



# Lecture 5: Finding Regulatory Motifs Within DNA Sequences

Study Chapter 4.4-4.9

# Initiating Transcription



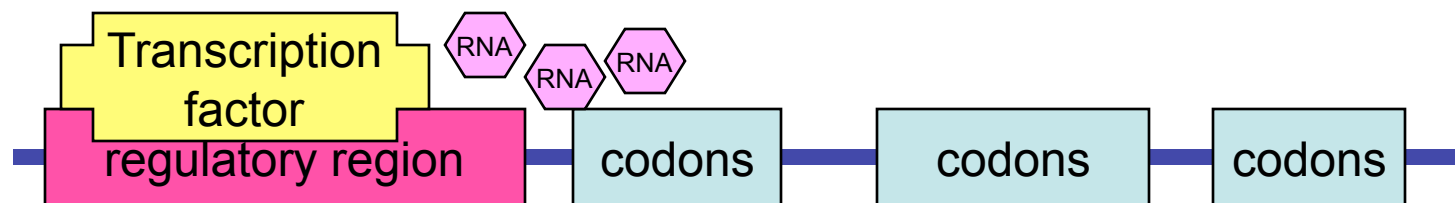
- As a precursor to transcription (the reading of DNA to construct RNAs that eventually leading to protein synthesis) special proteins bind to the DNA, separate it to enable its reading.
- How do these proteins know where the coding genes are in order to bind?
- Genes are relatively rare
  - $O(1,000,000,000)$  bases/genome
  - $O(10000)$  genes/genome
  - $O(1000)$  bases/gene
- Approximately 1% of DNA codes for genes ( $10^3 10^4 / 10^9$ )



# Regulatory Regions



- RNA polymerases seek out *regulatory* or *promoting* regions located 100-1000 bp upstream from the coding region
- They work in conjunction with special proteins called *transcription factors* whose presence enables gene expression
- Within these regions are the *Transcription Factor Binding Sites* (TFBS), special DNA sequence patterns known as *motifs* that are specific to a given transcription factor



# Transcription Factor Binding Sites



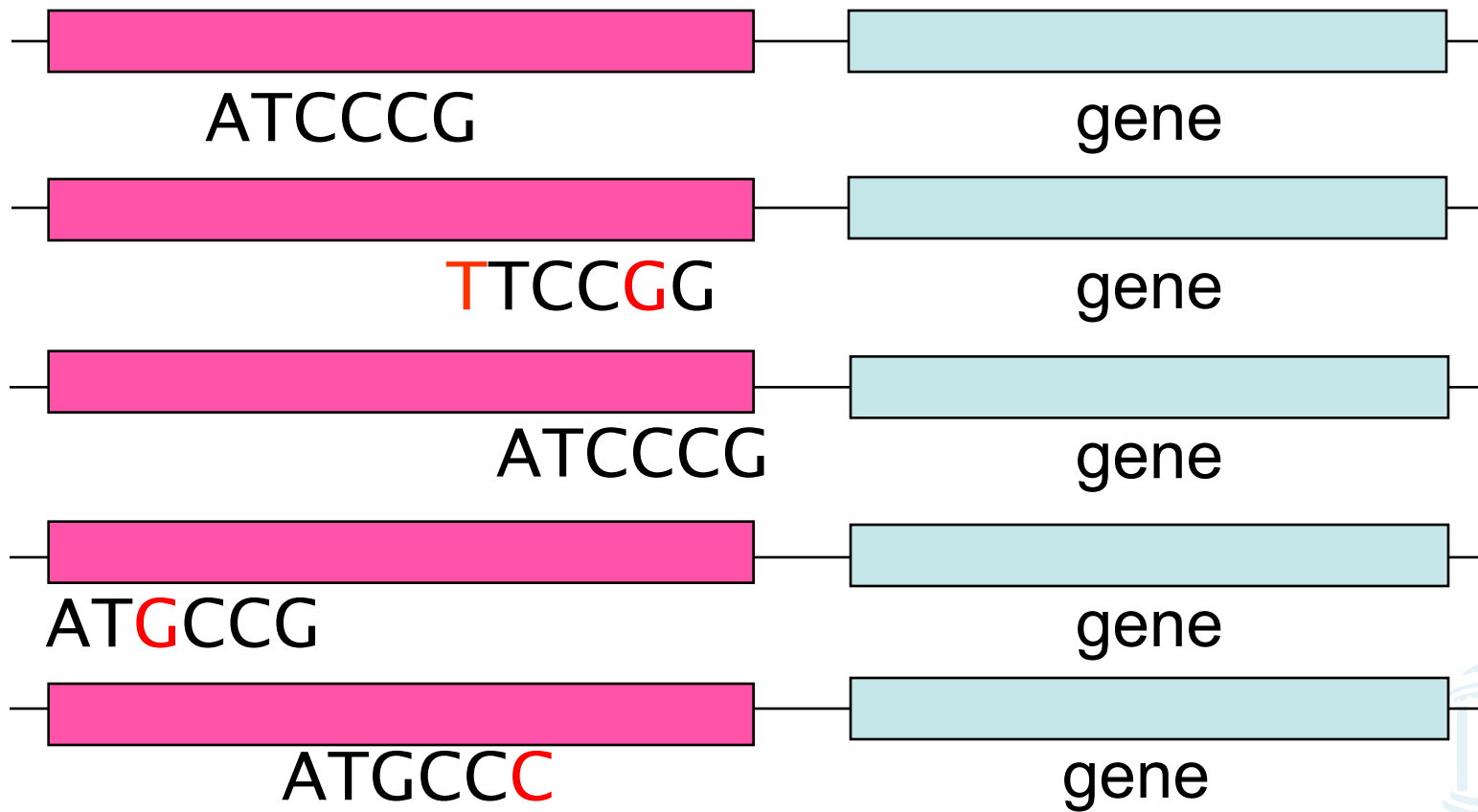
- A TFBS can be located anywhere within the regulatory region.
- TFBS may vary slightly across different regulatory regions since non-essential bases could mutate
- Transcription factors are robust (they will still bind) in the presence of small changes in a few bases



# Motifs and Transcriptional Start Sites



*Motif (n)* - A repeated structural element in architecture or decoration



# Identifying Motifs: Complications



- We do not know the motif sequence for every TF
- We do not know where it is located relative to the gene's start
- Motifs can differ slightly from one gene to the next
- We only know that it occurs frequently
- How to discern a Motif's frequent "similar" pattern from "random" patterns?



# An Aside: Solving Cryptograms



- A popular form of word puzzle

N oucgupju dlgw ynouo nwu sbu ynoho ld n jlzu dlw eupuo, xbhjb, snqup hp swhvmuo, zusuwihpuo vwlsuhp oucgupjuo.

- Based on letter, multi-letter, and word frequency it is not hard to figure out the most likely answer.

- Try solving it using

<http://rumkin.com/tools/cipher/cryptogram-solver.php>



# How's a Motif Like a Cryptogram?



- Nucleotides in motifs encode a message in a “genetic” language. Symbols in a cryptogram, encode messages in English
- In order to solve the problem, we analyze the *frequencies of patterns* in DNA/Cryptogram.
- Knowledge of established regulatory motifs makes the Motif Finding problem simpler. Knowledge of the words in the English dictionary helps to solve cryptograms.





# Motif Finding Complications



- We don't have a complete dictionary of motifs
- The "genetic" language doesn't have a standard "grammar"
- Only a small fraction of nucleotide sequences encode for motifs
- The size of the genome sequence is enormous



# The Motif Finding Problem



- Given a random sample of DNA sequences:

```
cctgatagacgctatctggctatccacgtacgtaggtcctctgtgccaatctatgcgtttccaacat
agtactggtgtacatttgatacgtacgtacaccggcaacctgaaacaaacgctcagaaccagaagtgc
aacgtacgtgcaccctctttcttcgtggctctggccaacgagggctgatgtataagacgaaaatttt
agcctccgatgtaagtcatacgtgtaactattacctgccaccctattacatcttacgtacgtataca
ctgttatacaacgcgctcatggcggggtatgcgttttggctcgtcgtacgctcgatcgttaacgtacgtc
```

- Find the pattern that is implanted in each of the individual sequences, namely, the motif
- Additional information:
  - Assume the hidden sequence is of length 8
  - The pattern is not exactly the same in each sequence because random point mutations have been introduced



# Motif Finding Example



- Finding motifs if there are no mutations
- Probability of a given 8-mer in an infinite sequence is  $1/4^8 \approx 1.5 \times 10^{-5}$  (1 every 65Kb)
- Assuming 5 strings of length 68, there are  $5(68 - 8) = 300$  distinct 8-mers
- Probability of any one 8-mer is  $300/4^8 \approx 0.005$
- So *any* repeat is rare

cctgatagacgctatctggctatccacgtacgtaggctcctctgtgccaatctatgcgtttccaaccat  
agtactgggtgtacatttgatacgtacgtacaccggcaacctgaaacaaacgctcagaaccagaagtgc  
aaacgtacgtgcaccctctttcttcgtggctctggccaacgagggctgatgtataagacgaaaatttt  
agcctccgatgtaagtcatagctgtaactattacctgccaccctattacatcttacgtacgtataca  
ctgttatacaacgcgctcatggcggggtatgcgttttggtcgctcgctcgatcgttaacgtacgtc

**acgtacgt**



# The Problem Becomes Harder



- Introduce 2 point mutations into each pattern:

```
cctgatagacgctatctggctatccaGgtacTtaggtcctctgtgCGaatctatgcgtttccaacat
agtactgggtgtacatttgatCcAtacgtagaccggcaacctgaaacaaacgctcagaaccagaagtgc
aaacgtTAGtgaccctctttcttcgtggctctggccaacgagggctgatgtataagacgaaaatttt
agcctccgatgtaagtcatagctgtaactattacctgccaccctattacatcttacgtCCAtataca
ctgttatacaacgcgtcatggcggggatgCGttttggtcgctcgtacgctcgatcgттаCcgtagcGc
```

- Our original target pattern no longer appears in any sequence!

**Can we still find the motif?**



# Defining a Motif



- To define a motif, let's assume that we know where the motif starts in each sequence
- The start positions can be represented as

$$s = [s_1, s_2, s_3, \dots, s_t]$$



# Motifs: Profiles and Consensus



Alignment

a	G	g	t	a	c	T	t
C	c	A	t	a	c	g	t
a	c	g	t	T	A	g	t
a	c	g	t	C	c	A	t
C	c	g	t	a	c	g	G

Profile

A	3	0	1	0	3	1	1	0
C	2	4	0	0	1	4	0	0
G	0	1	4	0	0	0	3	1
T	0	0	0	5	1	0	1	4

Consensus

A	C	G	T	A	C	G	T
---	---	---	---	---	---	---	---

- Line up the patterns by their start indexes

$$\mathbf{s} = (s_1, s_2, \dots, s_t)$$

- Construct a matrix profile with the frequencies of each nucleotide in columns
- Consensus nucleotide in each position has the highest score in column

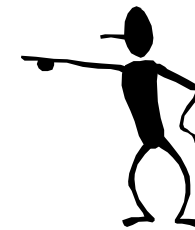


# Consensus



- One can think of the consensus as an “ancestor” motif, from which mutated motifs emerged
- The *distance* between an actual motif and the consensus sequence is generally less than that for any two actual motifs
- *Hamming distance* is number of positions that differ between two strings

<b>G</b>	A	G	A	C	<b>T</b>	C	A	T
X					X			
<b>T</b>	A	G	A	C	<b>G</b>	C	A	T



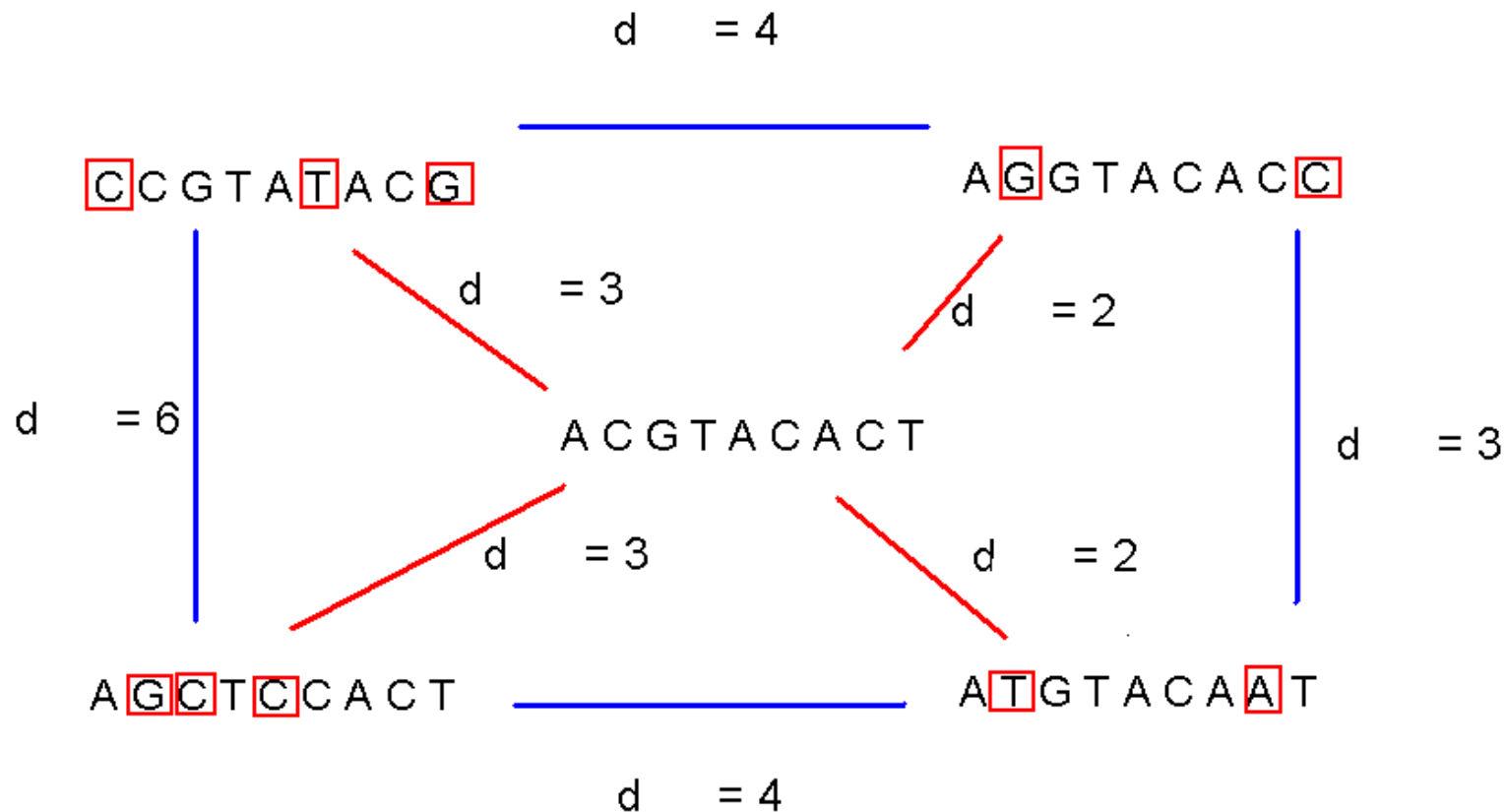
A Hamming  
distance of 2



# Consensus Properties



- A consensus string has a minimal hamming distance to all source strings





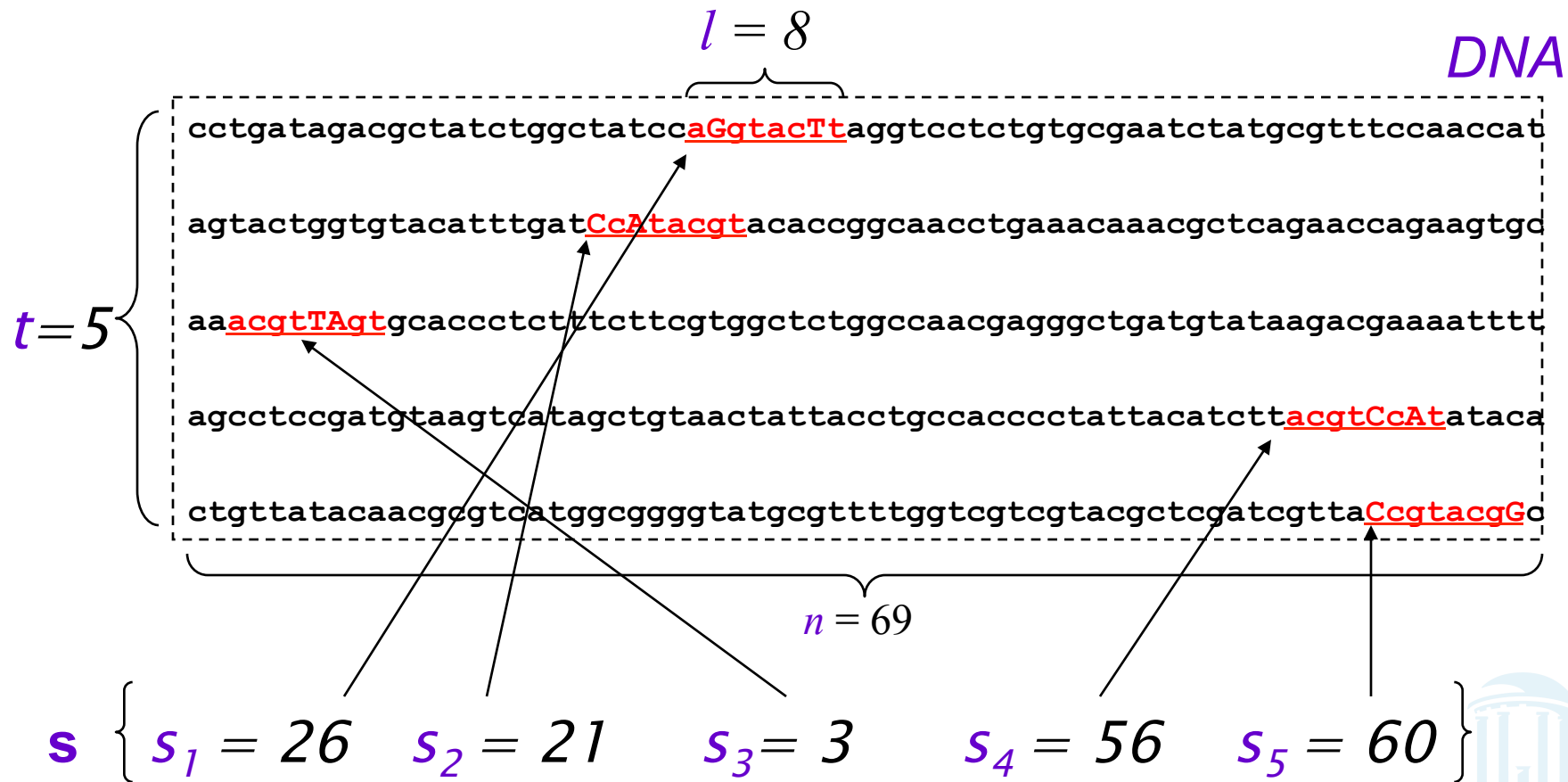
# Defining Some Terms



- *DNA* - array of sequence fragments
- *t* - number of sample DNA sequences
- *n* - length of each DNA sequence
  
- *ℓ* - length of the motif (*ℓ*-mer)
- $s_i$  - starting position of an *ℓ*-mer in sequence *i*
- $\mathbf{s} = (s_1, s_2, \dots, s_t)$  - array of motif's starting positions



# Illustration of Terms



# Scoring Motifs



- Given  $\mathbf{s} = (s_1, \dots, s_t)$  and *DNA*:

								$l$	
	a	G	g	t	a	c	T	t	}
	C	c	A	t	a	c	g	t	
	a	c	g	t	T	A	g	t	
	a	c	g	t	C	c	A	t	
	C	c	g	t	a	c	g	G	

$Score(\mathbf{s}, DNA) =$

$$\sum_{i=1}^l \text{Max}_{k \in \{A, C, G, T\}} \text{count}(k, i)$$

A	3	0	1	0	3	1	1	0
C	2	4	0	0	1	4	0	0
G	0	1	4	0	0	0	3	1
T	0	0	0	5	1	0	1	4

Consensus a c g t a c g t

Score 3+4+4+5+3+4+3+4=30



# The Motif Finding Problem



- Goal: Given a set of DNA sequences, find a set of  $\ell$ -mers, one from each sequence, that maximizes the consensus score
- Input: A  $t \times n$  matrix of *DNA*, and  $\ell$  the length of the pattern to find
- Output: An array of  $t$  starting positions  $\mathbf{s} = (s_1, s_2, \dots, s_t)$  maximizing  $Score(\mathbf{s}, DNA)$



# Brute Force Solution



- Compute the scores for all possible combinations of starting positions  $\mathbf{s}$
- The best score determines the best profile and the consensus pattern in  $DNA$
- The goal is to maximize  $Score(\mathbf{s}, DNA)$  by varying the starting positions  $s_i$ , where:

$$s_i = [1, \dots, n-l+1]$$
$$i = [1, \dots, t]$$



# Brute Force Pseudocode



1. BruteForceMotifSearch(DNA, t, n, l)
2. bestScore  $\leftarrow$  0
3. for each  $s = (s_1, s_2, \dots, s_t)$  from  $(1, 1, \dots, 1)$   
to  $(n-l+1, n-l+1, \dots, n-l+1)$
4. if  $\text{score}(s, \text{DNA}, l) > \text{bestScore}$
5.     bestScore  $\leftarrow$   $\text{score}(s, \text{DNA}, l)$
6.     bestMotif  $\leftarrow (s_1, s_2, \dots, s_t)$
7. return bestMotif



# Running Time of BruteForceMotifSearch



- Search  $(n - \ell + 1)$  positions in each of  $t$  sequences, by examining  $(n - \ell + 1)^t$  sets of starting positions
- For each set of starting positions, the scoring function makes  $\ell$  operations, so complexity is  $\ell(n - \ell + 1)^t = O(\ell n^t)$
- That means that for  $t = 8$ ,  $n = 1000$ ,  $\ell = 10$  we must perform approximately  $10^{25}$  computations
- Generously assuming  $10^9$  comps/sec it will require only  $10^{16}$  secs
- $10^{16} / (60 * 60 * 24 * 365) \rightarrow$  millions of years



# The Median String Problem



- Given a set of  $t$  DNA sequences find a pattern that appears in *all*  $t$  sequences with the minimum number of mutations
- This pattern will be the motif
- Rather than finding the maximal consensus string, this approach attempts to the minimal distance string

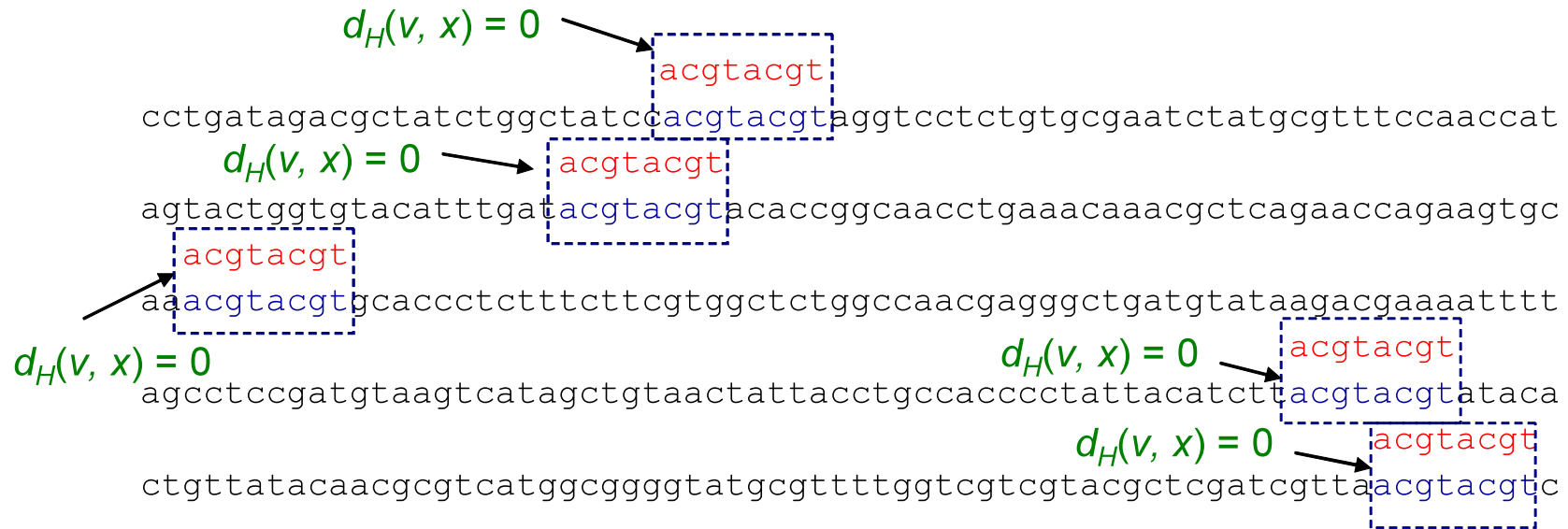




# Total Distance: An Example



- Given  $v = \text{"acgtacgt"}$  and  $s$



$v$  is the sequence in red,  $x$  is the sequence in blue

- $TotalDistance(v, DNA) = 0$



# Total Distance: An Example



- Given  $v = \text{"acgtacgt"}$  and  $s$

$d_H(v, x) = 1$  → acgtacg  
 cctgatagacgctatctggctatccacgtacAaggtcctctgtgCGaatctatgCGtTTccaacCAT

$d_H(v, x) = 0$  → acgtacgt  
 agtactggTgtacatttgatacgtacgtacaccggcaacCTgaaacaaacgctcagaaccagaagtgc

$d_H(v, x) = 2$  → acgtacgt  
 aaaAgCcgtgcaccctcttctctcgtggctctggccaacgagggctgatgtataagacgaaaattt

$d_H(v, x) = 0$  → acgtacgt  
 agcctccgatgtaagtcatagctgtaactattacctgccaccctattacatctacgtacgtataca

$d_H(v, x) = 1$  → acgtacgt  
 ctgttatacaacgcgctcatggcggggtatgCGtTTTggTCgtcgtacgctcgatcgTTAgTc

$v$  is the sequence in red,  $x$  is the sequence in blue

- $TotalDistance(v, DNA) = 1 + 0 + 2 + 0 + 1 = 4$



# Total Distance: Definition



- For each DNA sequence  $i$ , compute all  $d_H(v, x)$ , where  $x$  is an  $\ell$ -mer with starting position  $s_i$   
( $1 \leq s_i \leq n - \ell + 1$ )
- Find minimum of  $d_H(v, x)$  among all  $\ell$ -mers in sequence  $i$
- $TotalDistance(v, DNA)$  is the sum of the minimum Hamming distances for each DNA sequence  $i$
- $TotalDistance(v, DNA) = \min_s d_H(v, s)$ , where  $s$  is the set of starting positions  $s_1, s_2, \dots, s_t$



# The Median String Problem



- Goal: Given a set of DNA sequences, find a median string
- Input: A  $t \times n$  matrix  $DNA$ , and  $\ell$  the length of the pattern to find
- Output: A string  $v$  of  $\ell$  nucleotides that **minimizes**  $TotalDistance(v, DNA)$  over all strings of that length



# Median String Search Algorithm



1. MedianStringSearch(DNA, t, n, l)
2. bestMotif  $\leftarrow$  “”
3. bestDistance  $\leftarrow t \times l$
4. for each  $l$ -mer,  $s$ , from “aaa...a” to “ttt...t”
5. if TotalDistance( $s$ , DNA) < bestDistance
6.     bestDistance  $\leftarrow$  TotalDistance( $s$ , DNA)
7.     bestMotif  $\leftarrow s$
8. return bestMotif



# Equivalent Problems



- Motif Finding Problem  $\equiv$  Median String Problem
- The *Motif Finding* is a maximization problem while *Median String* is a minimization problem
- However, the *Motif Finding* problem and *Median String* problem are computationally equivalent (they give the same output for a common input)
- Need to show that minimizing *TotalDistance* is equivalent to maximizing *Score*



# We're looking for the same thing



Alignment

	{	a	G	g	t	a	c	T	t	
		C	c	A	t	a	c	g	t	
		a	c	g	t	T	A	g	t	
		a	c	g	t	C	c	A	t	
		C	c	g	t	a	c	g	G	

Profile

<b>A</b>	3	0	1	0	3	1	1	0
<b>C</b>	2	4	0	0	1	4	0	0
<b>G</b>	0	1	4	0	0	0	3	1
<b>T</b>	0	0	0	5	1	0	1	4

Consensus a c g t a c g t

**Score** 3+4+4+5+3+4+3+4

**TotalDistance** 2+1+1+0+2+1+2+1

**Sum** 5 5 5 5 5 5 5 5

- At any column  $i$   
 $Score_i + TotalDistance_i = t$
- Because there are  $\ell$  columns  
 $Score + TotalDistance = \ell * t$
- Rearranging:  
 $Score = \ell * t - TotalDistance$
- $\ell * t$  is constant the minimization of the right side is equivalent to the maximization of the left side



# Why Bother?



- What is the point of reformulating the Motif Finding problem as the Median String problem?
  - The Motif Finding Problem needs to examine all the combinations for  $s$ . That is  $(n - l + 1)^t$  combinations!!!
  - The Median String Problem needs to examine all  $4^l$  combinations for  $v$ . This number is relatively smaller

$$\begin{array}{l} n=1000, l=10, t=8 \\ \swarrow \searrow \\ (1000-10+1)^8 \approx 9.3 \times 10^{23} \\ 8(1000-10+1)4^{10} \approx 8.3 \times 10^9 \end{array}$$





# Improving Motif Finding



1. BruteForceMotifSearch(DNA, t, n, l)
2.  $\text{bestScore} \leftarrow 0$
3. for each  $s = (s_1, s_2, \dots, s_t)$  from  $(1, 1, \dots, 1)$   
to  $(n-l+1, n-l+1, \dots, n-l+1)$
4. if  $\text{score}(s, \text{DNA}, l) > \text{bestScore}$
5.      $\text{bestScore} \leftarrow \text{score}(s, \text{DNA}, l)$
6.      $\text{bestMotif} \leftarrow (s_1, s_2, \dots, s_t)$
7. return  $\text{bestMotif}$



# How to Structure the Search?



- How can we perform the line

for each  $s=(s_1, s_2, \dots, s_t)$  from  $(1, 1 \dots 1)$  to  $(n-l+1, \dots, n-l+1)$ ?

- We need a method to more efficiently examine the many possible motifs locations
- This is not very different than exploring all “ $t$ -digit base  $(n-l+1)$ ” numbers



# Improving Median String



1. MedianStringSearch(DNA, t, n, l)
2. bestMotif  $\leftarrow$  ""
3. bestDistance  $\leftarrow t \times l$
4. for each  $l$ -mer,  $s$ , from "aaa...a" to "ttt...t"
5. if TotalDistance( $s$ , DNA) < bestDistance
6.     bestDistance  $\leftarrow$  TotalDistance( $s$ , DNA)
7.     bestMotif  $\leftarrow s$
8. return bestMotif



# How to Best Explore Permutations?



- For the Median String Problem we need to consider all  $4^{\ell}$  possible  $\ell$ -mers:

aa... aa

aa... ac

aa... ag

aa... at

aa... ca

.

.

tt... tt

How to organize this search?



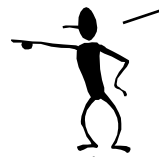
# Simple Code



- We've used variations of this idea before

```
def NextLeaf(a, L, k):  
    # generates L^k permutations  
    for i in reversed(xrange(L)):  
        if (a[i] < k):  
            a[i] += 1  
            break  
        else:  
            a[i] = 1  
    return a
```

The "break" here is very important is terminates the loop once any index is increased



- Each call generates a new permutation



# NextLeaf Usage



- This is the basic loop structure that we have used for many examples thus far (e. g. BruteForceChange)

```
def AllLeaves(L, k):  
    a = [1 for i in xrange(L)]  
    while True:  
        print a  
        a = NextLeaf(a, L, k)  
        if (sum(a) == L)  
            return
```

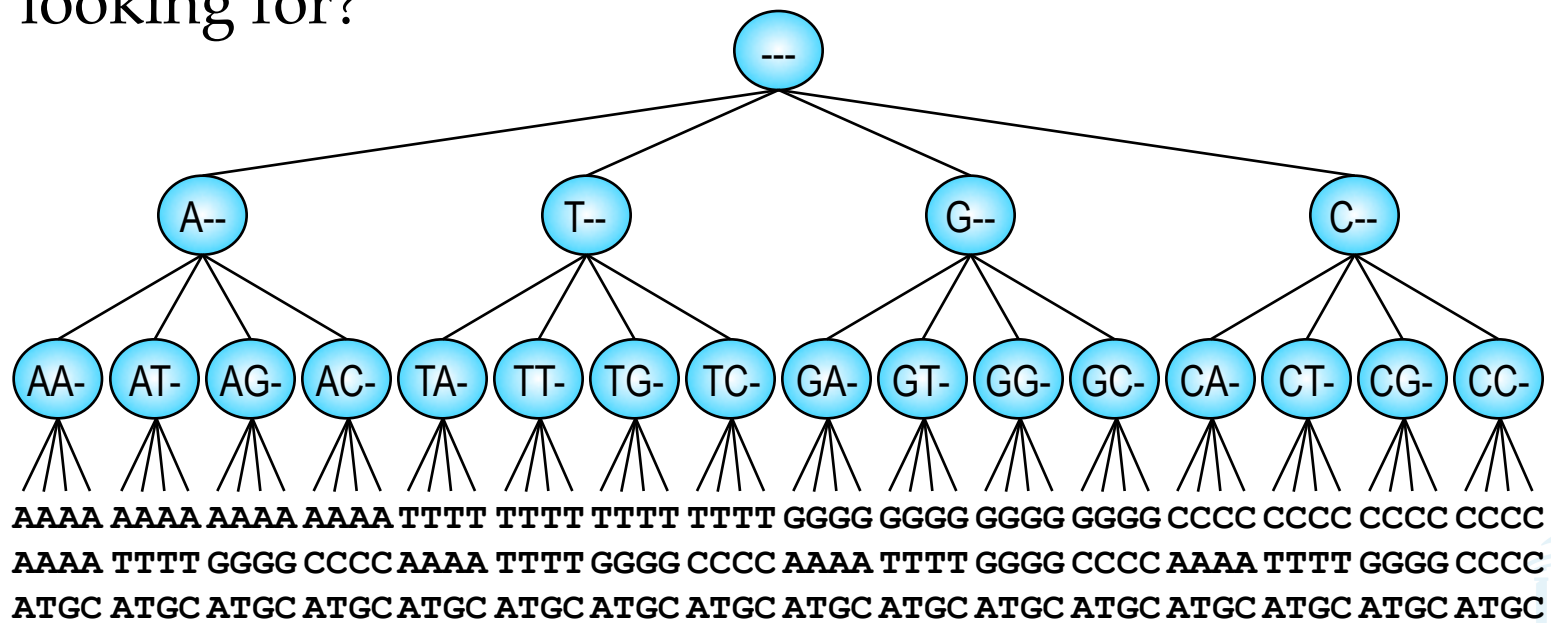
- Is there another way to search permutations?



# Search Tree



- Our standard method for enumerating permutations just traverses the leaf nodes
- Suppose after checking the first or second letter we already know the solution could not be the one we are looking for?



# Analyzing Search Trees



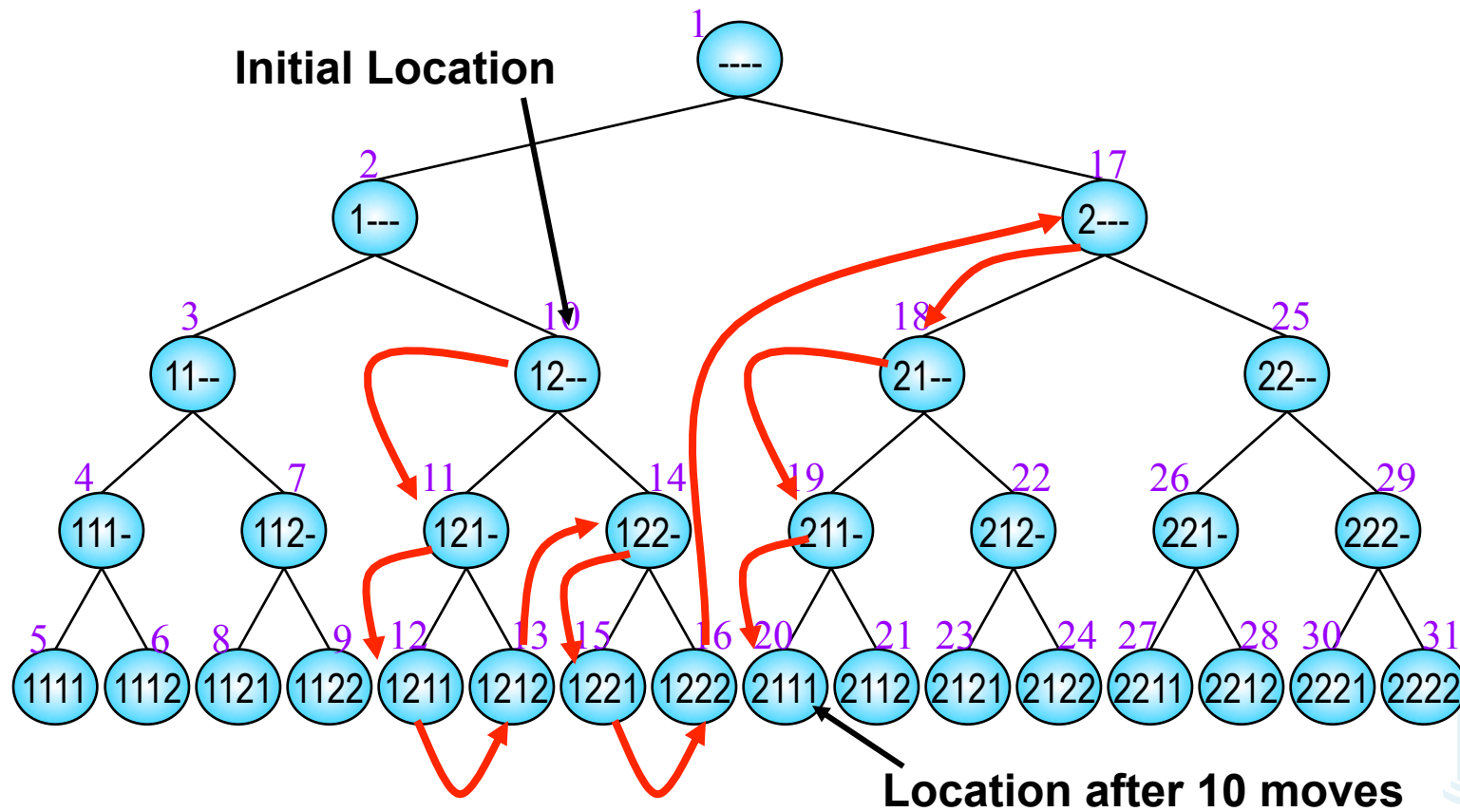
- Characteristics of the search trees:
  - The unique permutations reside at leaves
  - A parent node is a common prefix of its children
- How can we traverse the tree?
- Things we'd like to do:
  - Visit all the nodes (interior and leaves)
  - Visit the next node (in an ordered way)
  - Bypass the children of a node





# Depth First Search

- Start from the root and explore down to the bottom one path at a time



# Visiting the Next Vertex



- Uses 0s to encode unspecified part of interior nodes (the dashes in our figure)

```
def NextVertex(a, i, L, k):
    if (i < L):
        a[i] = 1
        return (a, i+1)
    else:
        for j in reversed(xrange(L)):
            if (a[j] < k):
                a[j] += 1
                return (a, j+1)
            a[j] = 0
    return (a, 0)
```



# Bypass Nodes



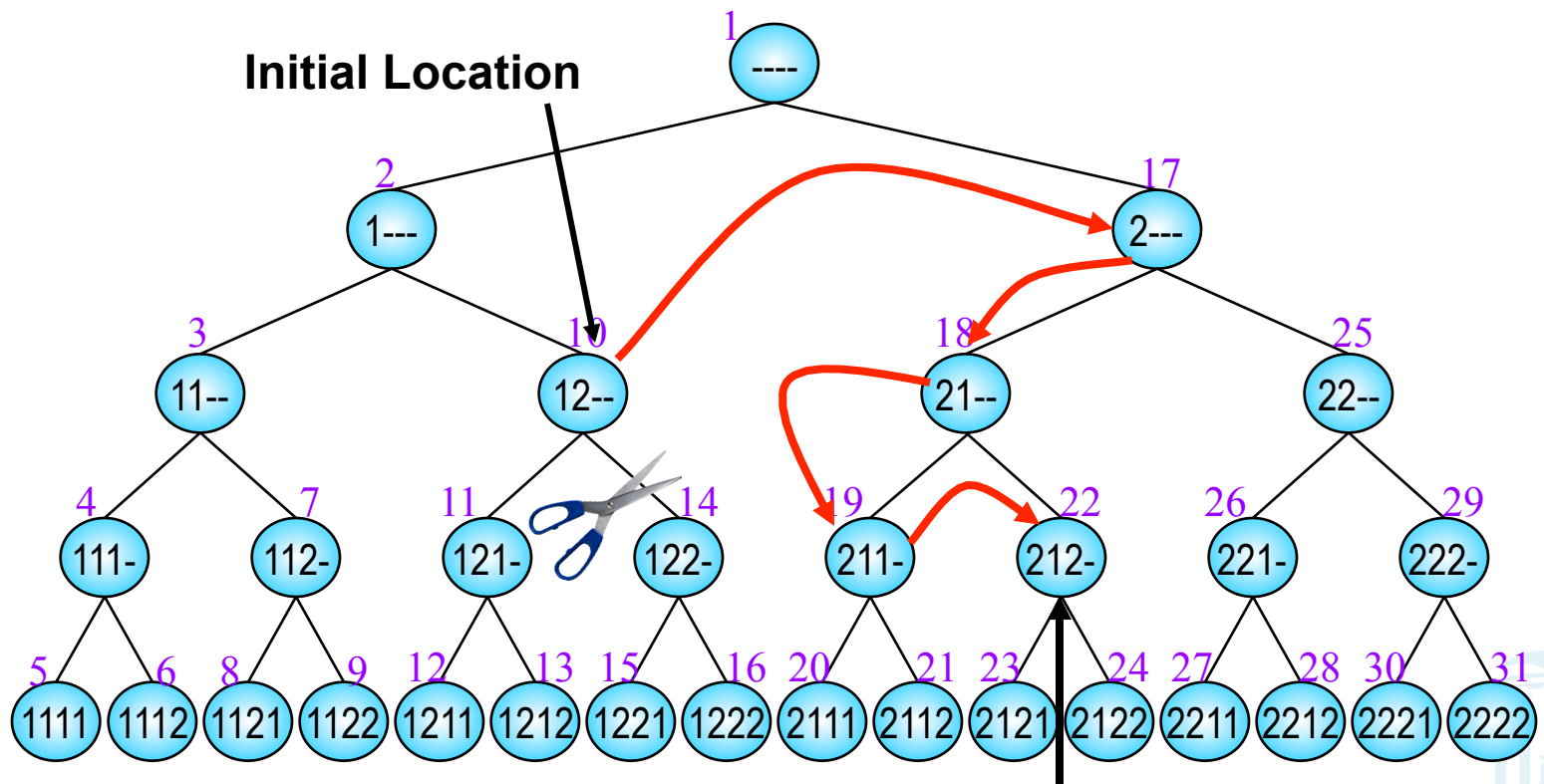
- Given a prefix (internal vertex), find next vertex after skipping all of the current vertex's children

```
def Bypass(a, i, L, k):  
    for j in reversed(xrange(i)):  
        if (a[j] < k):  
            a[j] += 1  
            return (a, j+1)  
        a[j] = 0  
    return (a, 0)
```



# Bypass Example

- Bypassing descendants of nodes “12–” and “211–”



# Revisiting Brute Force Search



- Now that we have method for navigating the tree, lets convert our pseudocode version of BruteForceMotifSearch to real code

```
def BruteForceMotifSearchAgain(DNA, t, n, l):  
    s = [1 for i in xrange(t)]  
    bestScore = Score(s, DNA)  
    while (True):  
        s = NextLeaf(s, t, n-1+1)  
        if (Score(s, DNA) > bestScore):  
            bestScore = Score(s, DNA)  
            bestMotif = [x for x in s]  
        if (sum(s) == t):  
            break  
    return bestMotif
```



# Can We Do Better?



- Sets of  $\mathbf{s}=(s_1, s_2, \dots, s_t)$  may have a weak profile for the first  $i$  positions  $(s_1, s_2, \dots, s_i)$
- Every row of alignment may add at most  $\ell$  to Score
- Optimism: if all subsequent  $(t-i)$  positions  $(s_{i+1}, \dots, s_t)$  add

$$(t - i) * \ell \text{ to } \text{Score}(\mathbf{s}, i, \text{DNA})$$

- If  $\text{Score}(\mathbf{s}, i, \text{DNA}) + (t - i) * \ell < \text{BestScore}$ , it makes no sense to search subtrees of the current vertex
  - Use **ByPass()**



# Rewrite Using Tree Traversal



- Before we apply a branch-and-bound strategy let's rewrite the brute-force algorithm using a search tree

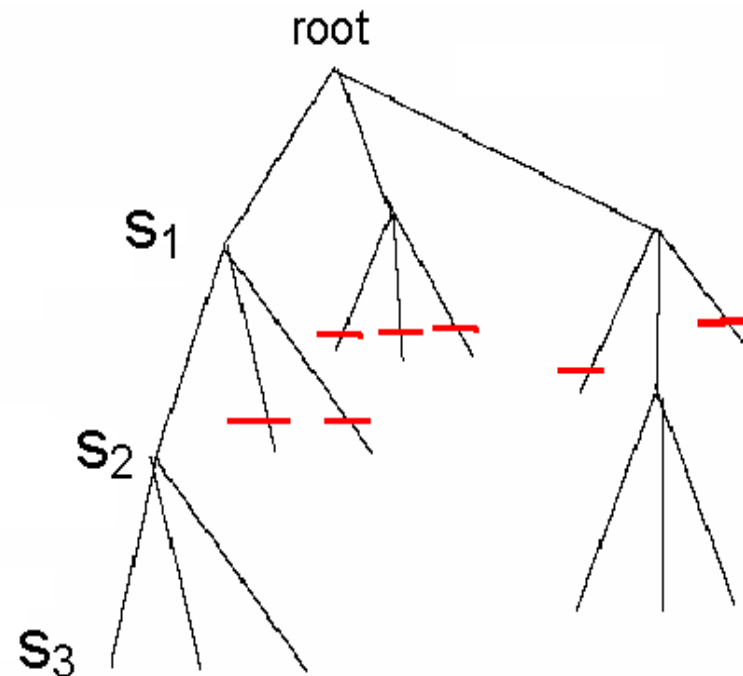
```
def SimpleMotifSearch(DNA, t, n, l):
    s = [0 for i in xrange(t)]
    bestScore = 0
    i = 0
    while (True):
        if (i < t):
            s, i = NextVertex(s, i, t, n-l+1)
        else:
            if (Score(s, DNA, l) > bestScore):
                bestScore = Score(s, DNA, l)
                bestMotif = [x for x in s]
            s, i = NextVertex(s, i, t, n-l+1)
            if (sum(s) == 0):
                break
    return bestMotif
```



# Branch and Bound Motif Search



- Since each level of the tree goes deeper into search, discarding a prefix discards all following branches
- This saves us from looking at  $(n - \ell + 1)^{t-i}$  leaves
  - Use **NextVertex()** and **ByPass()** to navigate the tree





# Branch-and-Bound Motif Code



```
def BranchAndBoundMotifSearch(DNA, t, n, l):
    s = [0 for i in xrange(t)]
    bestScore = 0
    i = 0
    while (True):
        if (i < t):
            optimisticScore = Score(s, DNA, l) + (t-i)*l
            if (optimisticScore < bestScore):
                s, i = Bypass(s, i, t, n-1+1)
            else:
                s, i = NextVertex(s, i, t, n-1+1)
        else:
            score = Score(s, DNA, l)
            if (score > bestScore):
                bestScore = score
                bestMotif = [x for x in s]
            s, i = NextVertex(s, i, t, n-1+1)
        if (sum(s) == 0):
            break
    return bestMotif
```



# Improving Median Search



- Recall the computational differences between motif search and median string search
  - The Motif Finding Problem needs to examine all  $(n-\ell+1)^\ell$  combinations for  $\mathbf{s}$ .
  - The Median String Problem needs to examine  $4^\ell$  combinations of  $\mathbf{v}$ . This number is relatively small
- We want to use median string algorithm with the Branch and Bound trick!



# Insight for Improving Median Search



- Note that if, at any point, the total distance for a prefix is greater than that for the best word so far:

$$\text{TotalDistance}(\text{prefix}, \text{DNA}) > \text{BestDistance}$$

there is no use exploring the remaining part of the word

- We can eliminate that branch and BYPASS exploring that branch further



# Bounded Median String Search



```
def BranchAndBoundMedianSearch(DNA, t, n, l):
    s = [1 for i in xrange(t)]
    bestDistance, bestWord = l*t, ''
    i = 1
    while (i > 0):
        if (i < l):
            prefix = NucleotideString(s, i)
            optimisticDistance = TotalDistance(prefix, DNA)
            if (optimisticDistance > bestDistance):
                s, i = Bypass(s, i, l, t)
            else:
                s, i = NextVertex(s, i, l, t)
        else:
            word = NucleotideString(s, l)
            if (TotalDistance(word, DNA) < bestDistance):
                bestDistance = TotalDistance(word, DNA)
                bestWord = word
            s, i = NextVertex(s, i, l, t)
    return bestWord
```



# Today's Bad Example



- An embarrassing confession. I got bitten by a bug in the online notes for the book!

```
cctgatagacgctatctggctatccaGgtacTtaggtcctctgtgCGaatctatgCGtttccaaccat
agtactgggtgtaCatttgatCCAtacgtacaccggcaacctgaaacaaacgctcagaaccagaagtgc
aaacgtTAgtgcaccctctttcttcgtggctctggccaacgagggctgatgtaataagacgaaaatTTT
agcctccgatgtagtcatagctgtaactattacctgccaccctattacatcttacctCCAtataca
ctgttatacaacgcgtcatggcggggatgCGttttggTCgTCgtacgctcGatcgTTaCCgtacgGC
```

- The target motif has a consensus score of 30
- But  $[2, 5, 46, 4, 1] = 31$  and  $[2, 5, 46, 6, 1] = 34$
- $>30$  solutions with consensus of 30 or better
- Which is the real Motif?



# Further Improvements



- More improvements to Motif searching
  - Why just prune based on prefixes?  
Can you consider suffixes too?
  - Consider a random subset of  $t$  strings, or  $l$  characters
  - Consider multiple letters at a time?
- How do you really find a TFBS?
  - Multiple answers
  - Near optimal answers
  - Motifs are just a starting point
- Next Time
  - We revisit greedy algorithms

