# Comp 555 - BioAlgorithms - Spring 2021



- WE'VE DISCUSSED

  SEQUENCING DNA, BUT

  THERE'S ANOTHER

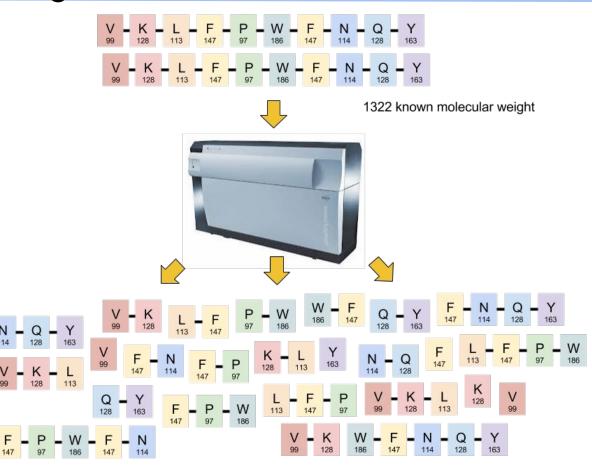
  SEQUENCE IN TOWN
- TO RECONSTRUCT IT, WE
  MUST FIRST BREAK IT
  INTO PIECES



Determining a Peptide's Sequence

## Molecular Weights are the Puzzle Peices





### Structure of a Peptide Chain



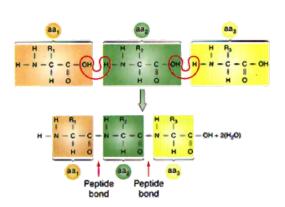
- Peptides are chains of amino acids that are joined by peptide bonds
- These bonds reduce the weight of each amino acid by one H<sub>2</sub>0 molecule
- The result is called a residue
- A Mass Spectrograph can precisely measure the molecular weight (and charge and abundance) of any peptide chain
- Since the molecular weight of each of the possible 20 residues is known precisely, one can ask the question, which combination of residues would give a particular weight?
- The problem is ambiguous for the entire molecule
  - Consider all permutations of 'PIT':

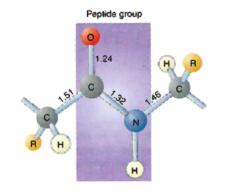
```
'PIT', 'PTI', 'ITP', 'IPT', 'TPI', and 'TIP' all weigh the same
```

But they differ in their 2-peptide fragments:

```
'PIT' breaks into 'PI' and 'IT', while 'PTI' breaks into 'PT' and 'TI'
```

 However, 'TIP' breaks into fragments 'TI' and 'IP', which have the same weights as 'PI' and 'TI' respectively. Thus, we can't tell 'PIT' from 'TIP'





## An Simplified Peptide Weight table



- The actual molecular weight of an amino acid is a real number.
   This accounts for the relative abundances of atomic isotopes
- Today, we will use a simplified version that assumes only integer molecular weights

#### Example:

Molecular weight of Glycine Amino Acid

$$W(C_2H_5NO_2) = 12\times2 + 5\times1 + 14 + 16\times2 = 75$$

Molecular weight of Glycine Residue (Minus the H<sub>2</sub>O lost forming the peptide bond)

$$W(C_2H_5NO_2-H_2O) = 57$$

 We can repeat this for all 20 Amino Acids to get a integer molecular weight table, which I name Daltons

### Table Definitions

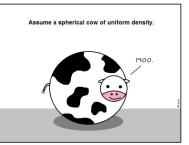


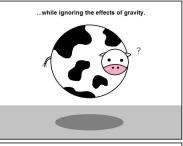
```
In [1]: AminoAcid = {
            'A': 'Alanine', 'C': 'Cysteine', 'D': 'Aspartic acid', 'E': 'Glutamic acid',
            'F': 'Phenylalanine', 'G': 'Glycine', 'H': 'Histidine', 'I': 'Isoleucine',
            'K': 'Lysine', 'L': 'Leucine', 'M': 'Methionine', 'N': 'Asparagine',
            'P': 'Proline', 'Q': 'Glutamine', 'R': 'Arginine', 'S': 'Serine',
            'T': 'Theronine', 'V': 'Valine', 'W': 'Tryptophan', 'Y': 'Tyrosine',
            '*': 'STOP'
        AminoAbbrv = {
            'A': 'Ala', 'C': 'Cys', 'D': 'Asp', 'E': 'Glu',
            'F': 'Phe', 'G': 'Gly', 'H': 'His', 'I': 'Ile',
            'K': 'Lys', 'L': 'Leu', 'M': 'Met', 'N': 'Asn',
            'P': 'Pro', 'Q': 'Gln', 'R': 'Arg', 'S': 'Ser',
            'T': 'Thr', 'V': 'Val', 'W': 'Trp', 'Y': 'Tvr',
            '*' 'STP'
        # Here's a new dictionary!
        Daltons = {
            'A': 71, 'C': 103, 'D': 115, 'E': 129,
            'F': 147, 'G': 57, 'H': 137, 'I': 113,
            'K': 128, 'L': 113, 'M': 131, 'N': 114,
            'P': 97, 'Q': 128, 'R': 156, 'S': 87,
            'T': 101, 'V': 99, 'W': 186, 'Y': 163
In [4]: averageMW = sum(Daltons.values())/20.0
        typicalLen = 1322/int(averageMW)
        print(averageMW, typicalLen, 20**typicalLen)
        118.75 11.203389830508474 376657155762813.56
```

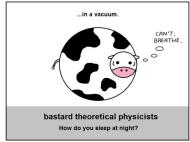
### Some Issues with our Table



- We can't distinguish between Leucine (L) and Isoleucine (I).
   They both weight 113d
- Nor can we distinguish Lysine (K) and Glutamine (Q), which weigh 128d
- For long peptide chains >50, our errors can build up
- In reality, peptides can lose or gain one or more small molecules from their side chains and fractured peptide bonds
  - Gain Hydrogen ions (H, +1 Dalton)
  - Lose Water (H2O, -18 Daltons)
  - Lose Ammonia (NH3, -17 Daltons)
- This leads to measurements that vary around the ideal sums we assume
- Regardless of these caveats, let's keep going







### The total molecular weight of our target



```
In [5]: TyrocidineB1 = "VKLFPWFNQY"

# The weight of Tyrocidine B1
print(sum([Daltons[res] for res in TyrocidineB1]))

1322
```

- Generally, we will assume that the peptide's total molecular weight is known
- We will use it as a terminating condition for many of our algorithms that attempt to reconstruct the peptide sequence from a measured set of weights

### What weights should we expect?



- We will make the optimistic assumption that we will fracture our given peptide chain into all of its constituent parts
- For a 10 peptide chain

10 single peptides	9, 2-peptide chains	8, 3-peptide chains
7, 4-peptide chains	6, 5-peptide chains	5, 6-peptide chains
4, 7-peptide chains	3, 8-peptide chains	2, 9-peptide chains
1, 10-peptide chain		

- This gives an upper bound of  $\binom{11}{2} = 55$  molecular weights
- In reality both the peptide chains and their weights may not be unique
- The collection of all possible sub-peptide molecular weights from a peptide is called the peptide's Theoretical Spectrum

## Code for computing a Theoretical Spectrum



```
In [7]: def TheoreticalSpectrum(peptide):
    # Generate every possible fragment of a peptide
    spectrum = set()
    for fragLength in range(1,len(peptide)+1):
        for start in range(0,len(peptide)-fragLength+1):
            seq = peptide[start:start+fragLength]
            spectrum.add(sum([Daltons[res] for res in seq]))
    return sorted(spectrum)

print(TyrocidineB1)
    spectrum = TheoreticalSpectrum(TyrocidineB1)
    print(len(spectrum))
    print(spectrum)

VKLFPWFNQY
51
[97, 99, 113, 114, 128, 147, 163, 186, 227, 241, 242, 244, 260, 261, 283, 291, 333, 340, 357, 388, 389, 405, 430, 44
```

7, 485, 487, 543, 544, 552, 575, 577, 584, 671, 672, 690, 691, 738, 770, 804, 818, 819, 835, 917, 932, 982, 1031, 106

Notice there are distinct 51 weights, how many would you expect?

0, 1095, 1159, 1223, 1322]

### Fragments and their Spectrums



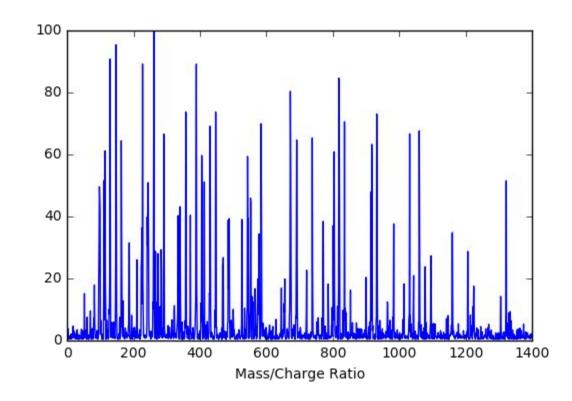
```
In [11]:
         peptide = TyrocidineB1
         fragList = []
         for fragLength in range(1,len(peptide)+1):
             for start in range(0,len(peptide)-fragLength+1):
                 seg = peptide[start:start+fragLength]
                 fragList.append((sum([Daltons[res] for res in seq]), seq))
         print(peptide)
         print(len(fragList))
         N = 0
         lastWeight = 0
         for weight, frag in sorted(fragList):
             print("%12s: %4d%s" % (frag, weight, "*" if (weight == lastWeight) else " "), end='')
             N += 1
             if (N \% 5 == 0):
                 print()
             lastWeight = weight
         VKLFPWFNQY
         55
                    P:
                         97
                                            99
                                                              113
                                                                                 114
                                                                                                   128
                       128*
                                           147
                                                              147*
                                                                                 163
                                                                                                    186
                    0:
                       227
                                           241
                                                              242
                   VK:
                                      KL:
                                                         NQ:
                                                                            FP:
                                                                                 244
                                                                                               LF:
                                                                                                   260
                   FN:
                       261
                                      PW:
                                           283
                                                         QY:
                                                              291
                                                                            WF:
                                                                                 333
                                                                                              VKL: 340
                  LFP:
                        357
                                     KLF:
                                           388
                                                              389
                                                                                              FPW: 430
                                                        FNQ:
                                                                           NQY:
                                                                                 405
                  PWF:
                       430*
                                     WFN:
                                           447
                                                       KLFP:
                                                              485
                                                                          VKLF:
                                                                                 487
                                                                                             LFPW: 543
                 PWFN:
                       544
                                    FNQY:
                                           552
                                                       WFNQ:
                                                              575
                                                                          FPWF:
                                                                                 577
                                                                                            VKLFP:
                                                                                                    584
                KLFPW: 671
                                   PWFNQ:
                                           672
                                                      LFPWF:
                                                              690
                                                                         FPWFN:
                                                                                 691
                                                                                            WFNQY:
                                                                                                   738
               VKLFPW:
                       770
                                  LFPWFN:
                                          804
                                                     KLFPWF:
                                                              818
                                                                        FPWFN0:
                                                                                 819
                                                                                           PWFNOY:
                                                                                                  835
              VKLFPWF: 917
                                 KLFPWFN:
                                                                       FPWFNQY:
                                          932
                                                    LFPWFNQ:
                                                              932*
                                                                                 982
                                                                                         VKLFPWFN: 1031
                                LFPWFNQY: 1095
             KLFPWFNO: 1060
                                                  VKLFPWFNQ: 1159
                                                                     KLFPWFNQY: 1223
                                                                                       VKLFPWFNQY: 1322
```

### What a Mass Spectrum looks like



- Peaks appear at frequently occurring mass locations
- Y-axis indicates the relative abundance, sometimes called relative intensity
- The peaks roughly correspond
   To our mass numbers

[97, 99, 113, 114, 128, 147, 163, 186, 227, 241, 242, 244, 260, 261, 283, 291, 333, 340, 357, 388, 389, 405, 430, 447, 485, 487, 543, 544, 552, 575, 577, 584, 671, 672, 690, 691, 738, 770, 804, 818, 819, 835, 917, 932, 982, 1031, 1060, 1095, 1159, 1223, 1322]



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## Let's try a smaller example



```
In [13]: peptide = 'PLAY'
         spectrum = TheoreticalSpectrum(peptide)
         print(len(spectrum), spectrum)
         fragList = []
         for fragLength in range(1,len(peptide)+1):
             for start in range(0,len(peptide)-fragLength+1):
                 seq = peptide[start:start+fragLength]
                fragList.append((sum([Daltons[res] for res in seq]), seq))
         print(len(fragList))
         N = 0
         lastWeight = 0
         for weight, frag in sorted(fragList):
             print("%12s: %4d%s" % (frag, weight, "*" if (weight == lastWeight) else " "), end='')
             N += 1
             if (N \% 5 == 0):
                 print()
             lastWeight = weight
         10 [71, 97, 113, 163, 184, 210, 234, 281, 347, 444]
         10
                                                                          Y: 163
                                                      L: 113
                                                                                           LA: 184
                  PL: 210
                                AY: 234
                                                      PLA: 281
                                                                         LAY: 347
                                                                                           PLAY: 444
```

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## Can we Invert the Process of creating a Spectrum?

@ MARK ANDERSON

 In essence, the problem of inferring a peptide chain from the set of mass values reported by a Mass Spectrometer is the inverse of the code we just wrote

**Easy Problem:** Peptide Sequence → Spectrum

Hard Problem: Spectrum → Peptide Sequence

- Why is computing a spectrum from a peptide sequence easy? O(N<sup>2</sup>)?
- Why is computing a peptide sequence from a spectrum hard? O(?)



"I'm trying to back it up, but I can't find reverse."

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### How might you approach this problem?



- Can you think of a Brute-Force way of solving this problem?
- Here's one:
  - 1. For every peptide sequence with the target peptide's molecular weight
  - 2. Compute the sequence's Theoretical Spectrum
  - 3. If it matches the one given, report this peptide as a possible solution
- Which step in this algorithm is the hard part?
- How many peptides have a molecular weight of 1322?
  - 1. How long is the longest peptide under 1322 daltons?
  - 2. How short is the shortest peptide over 1322 daltons?



### A Brute-Force Attempt



```
In [16]: def PossiblePeptide(spectrum, prefix=''):
             """ Brute force method of generating all peptide sequences with a desired weight, the max of a given spectrum """
             global peptideList
             if (len(prefix) == 0):
                 peptideList = []
             current = sum([Daltons[res] for res in prefix])
             target = max(spectrum) # our target
             if (current == target):
                 peptideList.append(prefix)
             elif (current < target):</pre>
                 for residue in Daltons.kevs():
                     PossiblePeptide(spectrum, prefix+residue)
         def TestPeptides(candidateList, target):
             filteredList = []
             for peptide in candidateList:
                 candidateSpectrum = TheoreticalSpectrum(peptide)
                 if (candidateSpectrum == target):
                     filteredList.append(peptide)
             return filteredList
         spectrum = TheoreticalSpectrum('PLAY')
         %time PossiblePeptide(spectrum)
         print(len(peptideList), "candidates", "PLAY" in peptideList)
         %time matches = TestPeptides(peptideList, spectrum)
         print(matches, "PLAY" in matches)
         CPU times: user 3.84 s, sys: 13 ms, total: 3.85 s
         Wall time: 3.85 s
         3687 candidates True
         CPU times: user 80 ms, sys: 0 ns, total: 80 ms
         Wall time: 79.8 ms
         ['PIAY', 'PLAY', 'YAIP', 'YALP'] True
```

### Impressions?



- Not so bad for a first attempt, but how will it perform for longer peptides?
- We are getting the expected answer as well as answers with the indistinguishable amino acids substituted
- We are also getting the sequence reversed? Is this a surprise?
- We could code around this, but for today we'll just include the reversed peptide chain as a possible answer

#### Could we do better?

- The brute force method does not make good use of the spectrum it is given
- It only ever considers the largest mass value from this table
- How might we make use of the other values?

### Improving on Brute Force



- We could extend our prefix using only residues that appear in our spectrum
- The weight of every new prefix that we consider should also be in our spectrum.

Actual fragments: P, L, A, Y, PL, LA, AY, PLA, LAY, PLAY

```
Ι
Α
ΑT
            IΑ
                = LA
                        LA = LA
                                   PΙ
                                        = PL
    = LA
                                                YΑ
                                                    = AY
ATP = PLA
           IAP = PLA
                       LAP = PLA
                                  PIA = PLA \qquad YAI = LAY
ATPY = PLAY TAPY = PLAY
                        LAPY = PLAY
                                  PIAY = PLAY
                                                YAIP = PLAY
ATY = LAY TAY = LAY
                        LAY = LAY
                                                YAL = LAY
AIYP = PLAY
           IAYP = PLAY
                        LAYP = PLAY
                                                YALP = PLAY
                                   PL
    = LA
            ΙP
                = PL
                        LP = PL
                                        = PL
Αl
ALP = PLA IPA = PLA
                       LPA = PLA
                                  PLA = PLA
ALPY = PLAY
            IPAY = PLAY
                        LPAY = PLAY
                                   PLAY = PLAY
ALY = LAY
ALYP = PLAY
    = AY
ΑY
AYT = LAY
AYTP = PLAY
AYL = LAY
AYLP = PLAY
```

## Only a small change



```
In [19]: def ImprovedPossiblePeptide(spectrum, prefix=''):
             global peptideList
             if (len(prefix) == 0):
                 peptideList = []
             current = sum([Daltons[res] for res in prefix])
             target = max(spectrum)
             if (current == target):
                 peptideList.append(prefix)
             elif (current < target):</pre>
                 for residue in Daltons.kevs():
                     # make sure that this residue appears in our spectrum
                     if (Daltons[residue] not in spectrum):
                         continue
                     # make sure that adding this residue to the sequence we have so far appears in our spectrum
                     extend = prefix + residue
                     if (sum([Daltons[res] for res in extend]) not in spectrum):
                         continue
                     ImprovedPossiblePeptide(spectrum, extend)
         spectrum = TheoreticalSpectrum('PLAY')
         %time ImprovedPossiblePeptide(spectrum)
         print(len(peptideList), "PLAY" in peptideList)
         print(peptideList)
         %time matches = TestPeptides(peptideList, spectrum)
         print(matches, "PLAY" in matches)
         CPU times: user 1 ms, sys: 0 ns, total: 1 ms
         Wall time: 708 µs
         16 True
         ['AIPY', 'AIYP', 'ALPY', 'ALPY', 'AYIP', 'AYIP', 'IAPY', 'IAYP', 'IPAY', 'LAYP', 'LAYP', 'LAYP', 'PLAY', 'YAI
         P', 'YALP']
         CPU times: user 1 ms, sys: 0 ns, total: 1 ms
         Wall time: 537 µs
         ['PIAY', 'PLAY', 'YAIP', 'YALP'] True
```

### Impact of a small change



- Provides a HUGE performace difference
- Yet another example of Branch-and-Bound
- We improved both the enumeration and verification phases, but the difference was much more significant in the enumeration step

- There are still differences in the spectrums, yet every prefix was in the spectrum when we added it. What are we missing?
- Suffixes!

### We can do Even Better



All suffixes of each prefix that we consider should also be in our spectrum

```
In [21]: def UltimatePossiblePeptide(spectrum, prefix=''):
             global peptideList
             if (len(prefix) == 0):
                 peptideList = []
             current = sum([Daltons[res] for res in prefix])
             target = max(spectrum)
             if (current == target):
                 peptideList.append(prefix)
             elif (current < target):</pre>
                 for residue in Daltons.keys():
                     extend = prefix + residue
                     # test every new suffix created by adding this new reside
                     # Note: this includes the residue itself as the length 1 suffix
                     suffix = [extend[i:] for i in range(len(extend))]
                     for fragment in suffix:
                         if (sum([Daltons[res] for res in fragment]) not in spectrum):
                              break
                     else:
                         UltimatePossiblePeptide(spectrum, extend)
         spectrum = TheoreticalSpectrum('PLAY')
         %time UltimatePossiblePeptide(spectrum)
         print(len(peptideList), peptideList, "PLAY" in peptideList)
         %time matches = TestPeptides(peptideList, spectrum)
         print(matches, "PLAY" in matches)
         CPU times: user 1.1 ms, sys: 4 µs, total: 1.11 ms
         Wall time: 1.12 ms
         4 ['PIAY', 'PLAY', 'YAIP', 'YALP'] True
         CPU times: user 113 us, svs: 0 ns, total: 113 us
         Wall time: 123 µs
         ['PIAY', 'PLAY', 'YAIP', 'YALP'] True
```

- A little slower, but our list is pruned significantly
- All of theses have identical spectrums

### Now let's return to our Real peptide



```
spectrum = TheoreticalSpectrum(TyrocidineB1)
In [23]:
         %time UltimatePossiblePeptide(spectrum)
         print(len(peptideList))
         print(TyrocidineB1 in peptideList)
         %time matches = TestPeptides(peptideList, spectrum)
         print(len(matches))
         print(TyrocidineB1 in matches)
         CPU times: user 31.4 ms, sys: 2.2 ms, total: 33.6 ms
         Wall time: 31.5 ms
         ['VKIFPWFNKY', 'VKIFPWFNQY', 'VKLFPWFNKY', 'VQIFPWFNKY', 'VQIFPWFNQY', 'VQLFPWFNQY', 'VQLFPWFNQY', 'YKN
         FWPFIKV', 'YKNFWPFIQV', 'YKNFWPFLKV', 'YONFWPFLQV', 'YONFWPFIKV', 'YONFWPFLKV', 'YONFWPFLKV', 'YONFWPFLQV']
         16
         True
         CPU times: user 1.11 ms, sys: 6 µs, total: 1.12 ms
         Wall time: 1.13 ms
         16
         True
In [24]: print(TyrocidineB1)
         for i, peptide in enumerate(peptideList):
             print(peptide, end=',')
             if (i % 4 == 3):
                 print()
         VKLFPWFNQY
         VKIFPWFNKY, VKIFPWFNQY, VKLFPWFNKY, VKLFPWFNQY,
         VQIFPWFNKY, VQIFPWFNQY, VQLFPWFNKY, VQLFPWFNQY,
         YKNFWPFIKV, YKNFWPFIQV, YKNFWPFLKV, YKNFWPFLQV,
         YONFWPFIKV, YONFWPFIOV, YONFWPFLKV, YONFWPFLOV,
```

### Great, but our assumptions are a little Naïve



- In reality, Mass Spectometers don't report the Theoretical Spectrum of a peptide
- Instead they report a measured or Experimental Spectrum
- This spectrum might *miss* some fragments
- It might also report false fragments
  - From Contaminants
  - New peptides formed by unintended reactions between fragments
- The result is that some of the masses that appear may be misleading, and some that we want might be missing
- We need to develop algorithms for reporting candidate protein sequences that are robust to noise

### MORE NEXT TIME