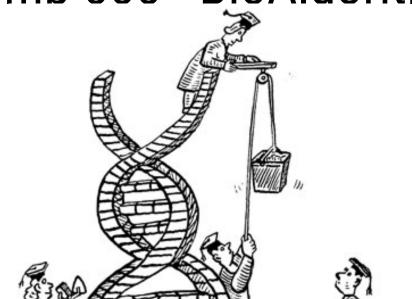
Comp 555 - BioAlaorithms - Spring 2020



PROBLEM SET #Z
 IS ON-LINE

The Realities of Genome Assembly

From Last Time

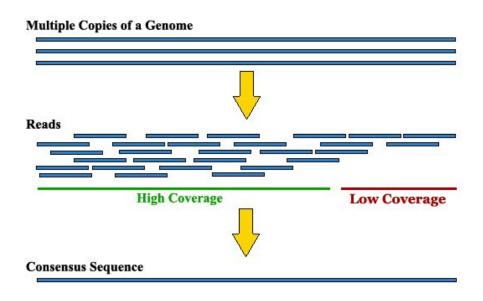


What we learned from a related "Minimal Superstring" problem

- Can be constructed by finding a Hamiltonian path of an k-dimensional De Bruijn graph over σ symbols
 - Brute-force method is explores all V! paths through V vertices
 - Branch-and-Bound method considers only paths composed of edges in the graph
 - Finding a Hamiltonian path is an NP-complete problem
 - There is no known method that can solve it efficiently as the number of vertices grows
- Can be solved by finding a Eulerian path of a (k-1)-dimensional De Bruijn graph where k-mers are edges.
 - Euler's method finds a path using all edges in $O(E) \le O(V^2)$ steps
 - Graph must statisfy contraints to be sure that a solution exists
- All but two vertices must be balanced
- The other two must be semi-balanced

Applications to Assembling Genomes





- Extracted DNA is broken into random small fragments
- 100-200 bases are read from one or both ends of the fragment
- Typically, each base of the genome is covered by 10x 30x fragments

Genome Assembly vs Minimal Superstring



```
binary3 = {'000', '001', '010', '011', '100', '101', '110', '111'}
```

```
101 100 111 100 001 101 Solution #1: 000101100 000 110 000 110 000 110 010 110 011 010
```

- Mininmal substring problem
 - Every k-mer is present and used as a vertex, (all σ^k)
 - o Paths, and there may be multiple, are solutions
- Read fragments
 - No guarantee that we will ever see every k-mer
 - Can't disambiguate repeats

Recall our "Toy" example



```
GACGGCGCGCACGGCGCAA
                        - Our toy 20 base sequence from 2 lectures ago
GACGG
        CGCAC
ACGGC
        GCACG
 CGGCG
          CACGG
                        - The complete set of 16 5-mers
   GGCGG
           ACGGC
   GCGGC
            CGGCG
     CGGCG
             GGCGC
      GGCGC
              GCGCA
       GGCGA
               CGCAA
```

Issues:

- All *k-mers* is equivalent to *k×* coverage, ignoring boundaries
- Four repeated k-mers {ACGGC, CGGCG, GCGCA, GGCGC}

Some Code



First let's add a function to uniquely label repeated k-mers

```
In [4]: M def kmersUnique(seq, k):
                kmers = sorted([seq[i:i+k] for i in range(len(seq)-k+1)])
                for i in range(1,len(kmers)):
                    if (kmers[i] == kmers[i-1][0:k]):
                        t = kmers[i-1].find(' ')
                        if (t >= 0):
                            n = int(kmers[i-1][t+1:]) + 1
                            kmers[i] = kmers[i] + " " + str(n)
                        else:
                            kmers[i-1] = kmers[i-1] + _1"
                            kmers[i] = kmers[i] + " 2"
                return kmers
            kmers = kmersUnique("GACGGCGCGCGCGCGCAA", 5)
            print(kmers)
            ['ACGGC 1', 'ACGGC 2', 'CACGG', 'CGCAA', 'CGCAC', 'CGGCG 1', 'CGGCG 2', 'CGGCG 3', 'GACGG', 'GCACG', 'GCGCA 1', 'GCGCA 2',
            'GCGGC', 'GGCGC 1', 'GGCGC 2', 'GGCGG']
```

Our Graph class from last lecture



```
In [25]: import itertools
          class Graph:
              def init (self, vlist=[
                  """ Initialize a Graph
                  self.index = {v:i for
                  self.vertex = {i:v for
                  self.edge = []
                  self.edgelabel = []
              def addVertex(self, label)
                  """ Add a labeled vert
                  index = len(self.index
                  self.index[label] = in
                  self.vertex[index] = 1
              def addEdge(self, vsrc, vd
                 """ Add a directed edg
                  Repeated edges are dis
                  e = (self.index[vsrc],
                  if (repeats) or (e not
                     self.edge.append(e
                     self.edgelabel.app
              def hamiltonianPath(self):
                 """ A Brute-force meth
                  Basically, all possibl
                  for edges. Since edges
                  made for *which* versi
                  for path in itertools.
                     for i in xrange(le
                          if ((path[i],p
                              break
                     else:
                          return [self.v
                  return []
              def SearchTree(self, path,
                  """ A recursive Branch
                  Paths are extended one
                  edges from the graph.
                  if (len(verticesLeft)
                     self.PathV2result
                     return True
                  for v in verticesLeft:
                     if (len(path) == 0
                          if self.Search
                              return Tru
                  return False
```

```
def hamiltonianPathV2(s
    """ A wrapper funct
    Hamiltonian Path se
   self.PathV2result =
    self.SearchTree([],
    return self.PathV2r
def degrees(self):
   """ Returns two dic
   of each node from t
    inDegree = {}
   outDegree = {}
    for src, dst in sel
        outDegree[src]
        inDegree[dst] =
    return inDegree, ou
def verifvAndGetStart(s
   inDegree, outDegree
   start = 0
    end = 0
    for vert in self.ve
        ins = inDegree.
        outs = outDegre
        if (ins == outs
            continue
        elif (ins - out
            end = vert
        elif (outs - in
            start = ver
        else:
            start, end
            break
    if (start >= 0) and
        return start
    else:
        return -1
def eulerEdges(self, pa
    edgeId = {}
    for i in xrange(len
        edgeId[self.edg
    edgeList = []
    for i in xrange(len
        edgeList.append
    return edgeList
```

```
def eulerianPath(self):
    graph = [(src,dst) for src,dst in
    currentVertex = self.verifyAndGet
    path = [currentVertex]
    # "next" is where vertices get in
    # it starts at the end (i.e. it i
    # but later "side-trips" will ins
    next = 1
    while len(graph) > 0:
        for edge in graph:
            if (edge[0] == currentVer
                currentVertex = edge[
                graph.remove(edge)
                path.insert(next, cur
                next += 1
                break
        else:
            for edge in graph:
                try:
                    next = path.index
                    currentVertex = e
                    break
                except ValueError:
                    continue
            else:
                print "There is no pa
                return False
    return path
def render(self, highlightPath=[]):
    """ Outputs a version of the grap
    using graphviz tools (http://www.
    edgeId = {}
    for i in xrange(len(self.edge)):
        edgeId[self.edge[i]] = edgeId
    edgeSet = set()
    for i in xrange(len(highlightPath
        src = self.index[highlightPat
        dst = self.index[highlightPat
        edgeSet.add(edgeId[src.dst].p
    result = ''
    result += 'digraph {\n'
    result += ' graph [nodesep=2, s
    for index, label in self.vertex.i
        result += ' N%d [shape="bo
```

```
for i, e in enumerate(self.edge):
    src, dst = e
    result += '
                  N%d -> N%d' % (src, dst)
    label = self.edgelabel[i]
    if (len(label) > 0):
       if (i in edgeSet):
            result += ' [label="%s", penwidth=3.0]' % (label)
            result += ' [label="%s"]' % (label)
    elif (i in edgeSet):
       result += ' [penwidth=3.0]'
    result += ':\n'
result += '
              overlap=false; \n'
result += '}\n'
return result
```

Finding Paths in our K-mer De Bruijn Graphs

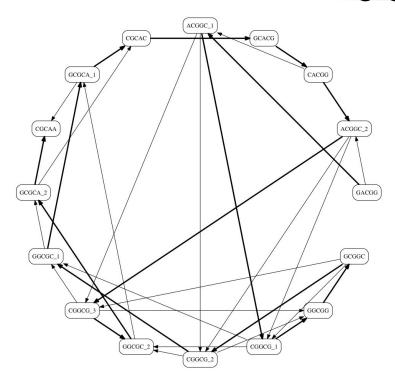


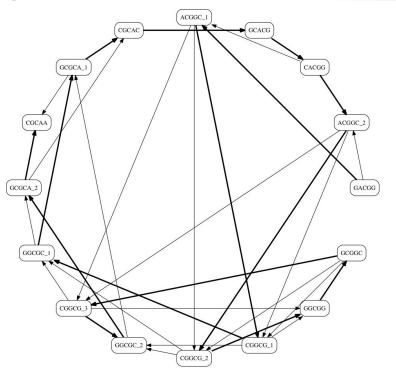
```
In [8]: N k = 5
            target = "GACGGCGCGCACGGCGCAA"
            kmers = kmersUnique(target, k)
            G1 = Graph(kmers)
            for vsrc in kmers:
                for vdst in kmers:
                    if (vsrc[1:k] == vdst[0:k-1]):
                        G1.addEdge(vsrc,vdst)
            path = G1.hamiltonianPathV2()
            print(path)
            seq = path[0][0:k]
            for kmer in path[1:]:
                seq += kmer[k-1]
            print(seq)
            print(seq == target)
            ['GACGG', 'ACGGC 1', 'CGGCG 1', 'GGCGC 1', 'GCGCA 1', 'CGCAC', 'GCACG', 'CACGG', 'ACGGC 2', 'CGGCG 2', 'GGCGG', 'GCGGC', 'CG
            GCG 3', 'GGCGC 2', 'GCGCA 2', 'CGCAA']
            GACGGCGCACGGCGCGCAA
            False
```

Not the sequence we expected ...

Let's look at the resulting graphs





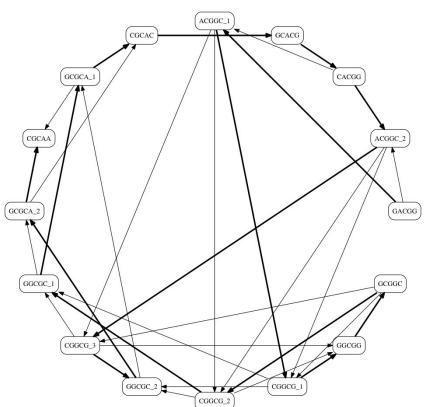


The one we hoped for. Visits CGGCG₃ before CGGCG₂

The one we found visits CGGCG_2 before CGGCG_3

What's the Problem?





- There are many possible Hamiltonian Paths
- How do they differ?
 - There were two possible paths leaving any [CGGCG] node
 - [CGGCG] → [GGCGC]
 - $\blacksquare \quad [\mathsf{CGGCG}] \to [\mathsf{GGCGG}]$
 - A valid solution can be found down either path
- There might be even more solutions
- Genome assembly appears ambiguous like the Minimal Substring problem, but is it?

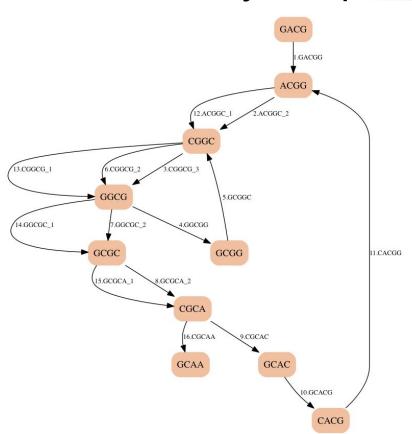
How about an Euler Path?



```
In [20]: N k = 5
             target = "GACGGCGCGCACGGCGCAA"
             kmers = kmersUnique(target, k)
             print(kmers)
             nodes = sorted(set([code[:k-1] for code in kmers] + [code[1:k] for code in kmers]))
             print(nodes)
             G2 = Graph(nodes)
             for code in kmers:
                G2.addEdge(code[:k-1],code[1:k],code)
             path = G2.eulerianPath()
             print(path)
             path = G2.eulerEdges(path)
             print(path)
             seg = path[0][0:k]
             for kmer in path[1:]:
                 seq += kmer[k-1]
             print(sea)
             print(seg == target)
             ['ACGGC 1', 'ACGGC 2', 'CACGG', 'CGCAA', 'CGCAC', 'CGGCG 1', 'CGGCG 2', 'CGGCG 3', 'GACGG', 'GCACG', 'GCGCA 1', 'GCGCA 2',
             'GCGGC', 'GGCGC 1', 'GGCGC 2', 'GGCGG']
             ['ACGG', 'CACG', 'CGCA', 'CGGC', 'GACG', 'GCAA', 'GCAC', 'GCGC', 'GCGG', 'GGCG']
             [4, 0, 3, 9, 8, 3, 9, 7, 2, 6, 1, 0, 3, 9, 7, 2, 5]
             ['GACGG', 'ACGGC 2', 'CGGCG 3', 'GGCGG', 'GCGGC', 'CGGCG 2', 'GGCGC 2', 'GCGCA 2', 'CGCAC', 'GCACG', 'CACGG', 'ACGGC 1', 'CG
             GCG 1', 'GGCGC 1', 'GCGCA 1', 'CGCAA']
             GACGGCGCGCACGGCGCAA
             True
```

The k-1 De Bruijn Graph with k-mer edges

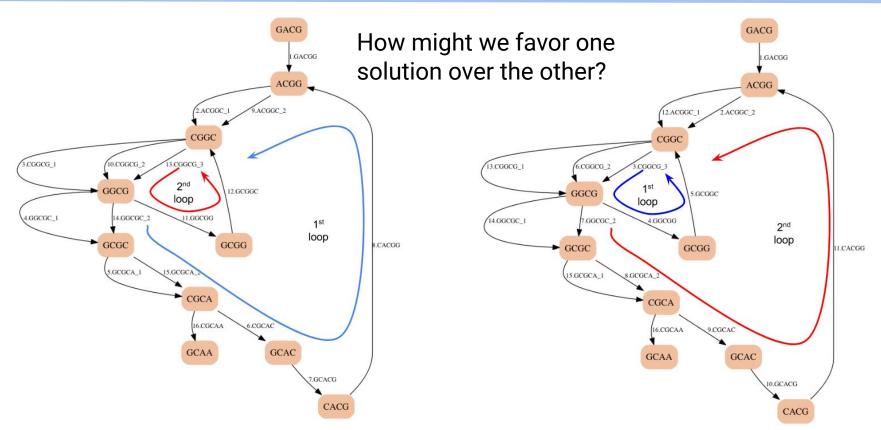




- We got the right answer, but we were lucky.
- There is a path in this graph that matches the Hamiltonian path that we found before

What are the Differences?





Choose a bigger k-mer

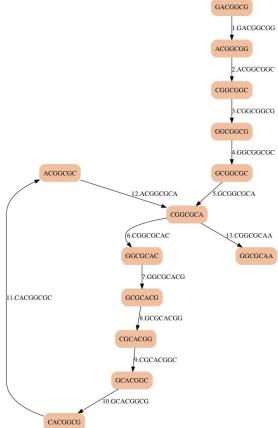


```
In [22]: | k = 8
                                       target = "GACGGCGCGCACGGCGCAA"
                                       kmers = kmersUnique(target, k)
                                        print(kmers)
                                       nodes = sorted(set([code[:k-1] for code in kmers] + [code[1:k] for code in kmers]))
                                        print(nodes)
                                       G3 = Graph(nodes)
                                       for code in kmers:
                                                 G3.addEdge(code[:k-1],code[1:k],code)
                                       path = G3.eulerianPath()
                                        print(path)
                                       path = G3.eulerEdges(path)
                                        print(path)
                                        seq = path[0][0:k]
                                        for kmer in path[1:]:
                                                    seq += kmer[k-1]
                                        print(seq)
                                        print(seq == target)
                                        ['ACGGCGCA', 'ACGGCGGC', 'CACGGCGC', 'CGCACGGC', 'CGGCGCAA', 'CGGCGCAC', 'CGGCGGCG', 'GACGGCGG', 'GCACGGCG', 'GCACGGCG', 'GCACGGC', 'GCACGGCG', 'GCACGGC', 'GCAC', 'GCACG
                                       GGCGCA', 'GGCGCACG', 'GGCGGCGC']
                                        ['ACGGCGC', 'ACGGCGG', 'CACGGCG', 'CGCACGG', 'CGGCGCA', 'CGGCGGC', 'GACGGCG', 'GCACGGC', 'GCGCACG', 'GCGCGCA', 'G
                                       GCGCAC', 'GGCGGCG']
                                       [6, 1, 5, 12, 9, 4, 11, 8, 3, 7, 2, 0, 4, 10]
                                        ['GACGGCGG', 'ACGGCGGC', 'CGGCGGCG', 'GGCGGCGC', 'GCGGCGCA', 'CGGCGCAC', 'GGCGCACG', 'GCGCACGG', 'CGCACGGC', 'GCACGGCG', 'CA
                                       CGGCGC', 'ACGGCGCA', 'CGGCGCAA']
                                       GACGGCGCGCACGGCGCAA
                                       True
```

Advantage of larger k-mers



- Making k larger (8) eliminates the second choice of loops
- There are edges to choose from, but they all lead to the same path of vertices



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Applied to the Hamiltonian Solution

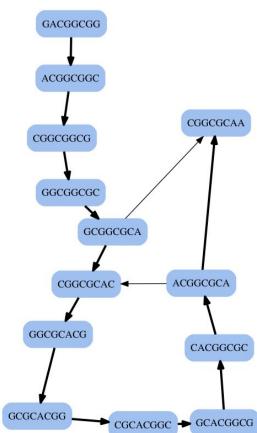


```
In [23]: N k = 8
             target = "GACGGCGCGCACGGCGCAA"
             kmers = kmersUnique(target, k)
             G4 = Graph(kmers)
             for vsrc in kmers:
                for vdst in kmers:
                    if (vsrc[1:k] == vdst[0:k-1]):
                        G4.addEdge(vsrc,vdst)
             path = G4.hamiltonianPathV2()
             print(path)
             seq = path[0][0:k]
             for kmer in path[1:]:
                seq += kmer[k-1]
             print(seq)
             print(seq == target)
             ['GACGGCGG', 'ACGGCGGC', 'CGGCGGCG', 'GCGGCGCC', 'GCGGCGCA', 'CGGCGCAC', 'GGCGCACG', 'GCGCACGG', 'CGCACGGC', 'CA
             CGGCGC', 'ACGGCGCA', 'CGGCGCAA']
             GACGGCGCGCACGGCGCAA
             True
```

Graph with 8-mers as vertices



- There is only one Hamiltonian path
- There are no repeated k-mers



Assembly in Reality



- Problems with repeated k-mers
- We can't distinguish between repeated k-mers
- Recall we knew from our example that were {2:ACGGC, 3:CGGCG, 2:GCGCA, 2:GGCGC}
- Assembling path without repeats:

```
In [26]: N k = 5
    target = "GACGGCGGCGCACGGCGCAA"
    kmers = set([target[i:i+k] for i in range(len(target)-k+1)])
    nodes = sorted(set([code[:k-1] for code in kmers] + [code[1:k] for code in kmers]))
    G5 = Graph(nodes)
    for code in kmers:
        G5.addEdge(code[:k-1],code[1:k],code)

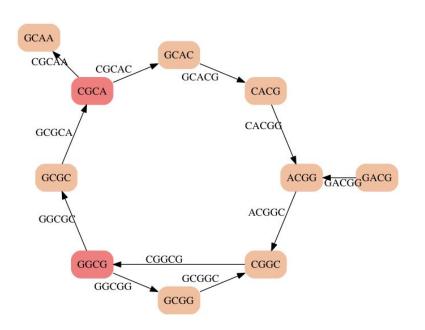
    print(sorted(G5.vertex.items()))
    print(G5.edge)

[(0, 'ACGG'), (1, 'CACG'), (2, 'CGCA'), (3, 'CGGC'), (4, 'GACG'), (5, 'GCAA'), (6, 'GCAC'), (7, 'GCGC'), (8, 'GCGG'), (9, 'GGGC')]
    [(9, 8), (3, 9), (1, 0), (4, 0), (6, 1), (8, 3), (0, 3), (2, 5), (7, 2), (2, 6), (9, 7)]
```

Resulting Graph with "unique" 5-mers as edges



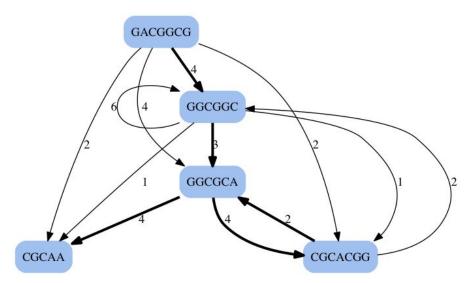
- There is no single Euler Path
- But there are is a set of paths that covers all edges ['GACGGCG', 'GGCGGC', 'GGCGCA', 'CGCAA', 'CGCACGG']
 - Extend a sequence from a node until you reach
 a node with an out-degree → in-degree
 - Save these partially assembled subsequences, call them *contigs*
 - Start new contigs following each out-going edge at these branching nodes



Next assemble contigs



- Use a modified read-overlap graph to assemble these contigs
- Add edge-weights that indicate the amount of overlap



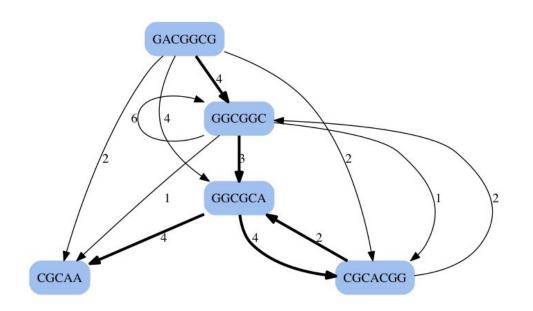
Usually much smaller than the graph made from k-mers

A Heavy Path



Find the heaviest path touching all vertices in this smaller graph

GACGGCGCACGGCGCAA GACGGCG	
GGCGGC	4
GGCGCA	3
CGCACGG	4
GGCGCA	2
CGCAA	4
	17



Discussion



- No simple single algorithm for assembling a real genome sequences
- Generally, an iterative task
 - Choose a k-mer size, ideally such that no or few k-mers are repeated
 - Assemble long paths (contigs) in the resulting graph
 - Use these contigs, if they overlap suffciently, to assemble longer sequences
- Truely repetitive subsequences are a challenge
 - Leads to repeated k-mers and loops in graphs in the problem areas
 - Often we assemble the "shortest" version of a genome consistent with our k-mer set
- Things we've ignored
 - Our k-mers are extracted from short read sequences that may contain errors
 - Our short read set could be missing entire segments from the actual genome
 - Our data actually supports 2 paths, one through the primary sequence, and a second through it again in reverse complement order.