## Comp 555 - BioAlaorithms - Spring 2020

- Problem set \#z is ON-LINE

The Realities of Genome Assembly

## From Last Time

## What we learned from a related "Minimal Superstring" problem

- Can be constructed by finding a Hamiltonian path of an k-dimensional De Bruijn graph over $\sigma$ symbols
- Brute-force method is explores all V! paths through V vertices
- Branch-and-Bound method considers only paths composed of edges in the graph
- Finding a Hamiltonian path is an NP-complete problem
- There is no known method that can solve it efficiently as the number of vertices grows
- Can be solved by finding a Eulerian path of a ( $\mathrm{k}-1$ )-dimensional De Bruijn graph where $k$-mers are edges.
- Euler's method finds a path using all edges in $\mathrm{O}(\mathrm{E}) \leq \mathrm{O}\left(\mathrm{V}^{2}\right)$ steps
- Graph must statisfy contraints to be sure that a solution exists
- All but two vertices must be balanced
- The other two must be semi-balanced


## Applications to Assembling Genomes



- Extracted DNA is broken into random small fragments
- 100-200 bases are read from one or both ends of the fragment
- Typically, each base of the genome is covered by $10 x-30 x$ fragments


## Genome Assembly vs Minimal Superstring

```
binary3 = {'000', '001', '010', '011', '100', '101', '110', '111'}
                    101 100
0 0 1 1 1 1
Solution #1: 0001011100
000 011
0 1 0 1 1 0
Solution #2: 0001110100
0 0 0 1 1 0
    0 1 1 0 1 0
```

- Mininmal substring problem
- Every k-mer is present and used as a vertex, (all $\sigma^{k}$ )
- Paths, and there may be multiple, are solutions
- Read fragments
- No guarantee that we will ever see every k-mer
- Can't disambiguate repeats


## Recall our "Toy" example



Issues:

- All $k$-mers is equivalent to $k \times$ coverage, ignoring boundaries
- Four repeated k-mers \{ACGGC, CGGCG, GCGCA, GGCGC\}


## Some Code

First let's add a function to uniquely label repeated $k$-mers

In [4]:

```
def kmersUnique(seq, k):
    kmers = sorted([seq[i:i+k] for i in range(len(seq)-k+1)])
    for i in range(1,len(kmers)):
        if (kmers[i] == kmers[i-1][0:k]):
            t = kmers[i-1].find('_')
            if ( }\textrm{t}>=0\mathrm{ ):
                n = int(kmers[i-1][t+1:]) + 1
                kmers[i] = kmers[i] + "_" + str(n)
            else:
            kmers[i-1] = kmers[i-1] + "_1"
            kmers[i] = kmers[i] + "_2"
    return kmers
kmers = kmersUnique("GACGGCGGCGCACGGCGCAA", 5)
print(kmers)
```

['ACGGC_1', 'ACGGC_2', 'CACGG', 'CGCAA', 'CGCAC', 'CGGCG_1', 'CGGCG_2', 'CGGCG_3', 'GACGG', 'GCACG', 'GCGCA_1', 'GCGCA_2',
'GCGGC', 'GGCGC_1', 'GGCGC_2', 'GGCGG']

## Our Graph class from last lecture

import itertools
class Graph
def _ninit__(self, vlist= Initialize a Graph self.index $=\{v: i$ for self.vertex $=\{i: v$ fo self.edge = self.edgelabel = []
def addVertex(self, label index = len(self. index self index[label] = in self.index[label] $=$ in ef addEdge(self, vsrc, vd f addEdge(self, vsrc, vd
""" Add a directed edg Repeated edges are dis
e = (self.index[vsrc],
if (repeats) or (e not self.edge.append ( self.edgelabel.app def hamiltonianPath(self) """ A Brute-force meth Basically, all possibl or edges. Since edges for path in itertools. for i in xrange(le if ( (path[i], p break
else: return []
def SearchTree(self, path """ A recursive Branch Paths are extended one edges from the graph. self.PathV2result selt.PathV2re return True
if (len(path) $==\theta$
if self.Search return Tru
return False
def hamiltonianPathv2(s A wrapper funct. Hamiltonian Path se self.PathV2result = self.SearchTree([], return self.PathV2r,
def degrees(self):
""" Returns two dic of each node from $t$ inDegree $=\{ \}$
outDegree $=\{$
for src, dst in sel outDegree[src] : inDegree[dst] = return inDegree, ou
def verifyAndGetStart(st inDegree, outDegree
start $=0$
end $=0$
for vert in self.ve ins = inDegree.: outs = outDegre
if (ins == outs continue elif (ins - out end = vert elif (outs - in start $=$ ver else:
start, end break
if (start >= 0) and return star els return -1
def eulerEdges(self, pa edgeId $=\{ \}$
for $i$ in xrange(len edgeId[self.edg. edgeList = []
for $i$ in xrange(len edgeList.append return edgeList
def eulerianPath(self)
graph $=[(s r c, d s t)$ for src, dst ir currentVertex $=$ self.verifyAndGet path $=$ [currentVertex]
\# "next" is where vertices get in \# it starts at the end (i.e. it $i$ \# but later "side-trips" will ins next $=1$
while len(graph) > 0
for edge in graph
if (edge[ $\theta$ ] = currentVer currentVertex $=$ edge[ graph.remove(edge) path.insert(next, cur next +=
break
else:
for edge in graph try: next = path.index currentVertex $=\epsilon$
except ValueError continue
else
print "There is no pa return False

## eturn path

def render(self, highlightPath=[]) "" Outputs a version of the grap sing graph dgeId $=\{ \}$
ge(len(self.edge)) edgeId[self.edge[i]] = edgeIc dgeSet $=$ set ()
or in in xrange(len(highlightPath dst = self.index[highlightPat edgeSet.add(edgeId[src,dst].F esult =
result += 'digraph \{\n'
result += ' graph [nodesep=2, s
for index, label in self.vertex.i result $+=\quad$ N\%d [shape="bc
for $i$, e in enumerate(self.edge):
sre, dst $=e$
result += N\%d -> N\%d' \% (src, dst
label = self.edgelabel[i]
if (len(label) $>0$ ):
result += ' [label="\%s", penwidth=3.0]' \% (label)
else:
result += ' [label="\%s"]' \% (label)
elif (i in edgeSet):
result $+=$. [penwidth=3.0]
result += '; $\ n$

```
result += ' overlap=false;\n
```

result += '\}
return result

## Finding Paths in our K-mer De Bruijn Graphs

In [8]: $\quad \mathrm{k}=5$
target $=$ "GACGGCGGCGCACGGCGCAA"
kmers $=$ kmersUnique(target, $k$ )
G1 $=$ Graph(kmers)
for vsrc in kmers
for vdst in kmers:
if $(\operatorname{vsrc}[1: k]==\operatorname{vdst}[0: k-1])$ :
G1.addEdge(vsrc, vdst)
path $=$ G1.hamiltonianPathV2()
print (path)
seq $=$ path[0][0:k]
for kmer in path[1:]:
seq $+=\operatorname{kmer}[k-1]$
print(seq)
print(seq == target)
['GACGG', 'ACGGC_1', 'CGGCG_1', 'GGCGC_1', 'GCGCA_1', 'CGCAC', 'GCACG', 'CACGG', 'ACGGC_2', 'CGGCG_2', 'GGCGG', 'GCGGC', 'CG GCG_3', 'GGCGC_2', 'GCGCA_2', 'CGCAA'] GACGGCGCACGGCGGCGCAA
False
Not the sequence we expected

## Let's look at the resulting graphs



The one we hoped for. Visits CGGCG $_{3}$ before CGGCG $_{2}$


The one we found visits CGGCG $_{2}$ before CGGCG $_{3}$

## What's the Problem?



- There are many possible Hamiltonian Paths
- How do they differ?
- There were two possible paths leaving any [CGGCG] node
- [CGGCG] $\rightarrow$ [GGCGC]
- [CGGCG] $\rightarrow$ [GGCGG]
- A valid solution can be found down either path
- There might be even more solutions
- Genome assembly appears ambiguous like the Minimal Substring problem, but is it?


## How about an Euler Path?

In [20]:

```
M k = 5
    target = "GACGGCGGCGCACGGCGCAA"
    kmers = kmersUnique(target, k)
    print(kmers)
    nodes = sorted(set([code[:k-1] for code in kmers] + [code[1:k] for code in kmers]))
    print(nodes)
    G2 = Graph(nodes)
    for code in kmers:
        G2.addEdge(code[:k-1], code[1:k],code)
    path = G2.eulerianPath()
    print(path)
    path = G2.eulerEdges(path)
    print(path)
    seq = path[0][0:k]
    for kmer in path[1:]:
        seq += kmer[k-1]
    print(seq)
    print(seq == target)
```

    ['ACGGC_1', 'ACGGC_2', 'CACGG', 'CGCAA', 'CGCAC', 'CGGCG_1', 'CGGCG_2', 'CGGCG_3', 'GACGG', 'GCACG', 'GCGCA_1', 'GCGCA_2',
    'GCGGC', 'GGCGC_1', 'GGCGC_2', 'GGCGG']
    ['ACGG', 'CACG', 'CGCA', 'CGGC', 'GACG', 'GCAA', 'GCAC', 'GCGC', 'GCGG', 'GGCG']
    [4, 0, 3, 9, 8, 3, 9, 7, 2, 6, 1, 0, 3, 9, 7, 2, 5]
    ['GACGG', 'ACGGC_2', 'CGGCG_3', 'GGCGG', 'GCGGC', 'CGGCG_2', 'GGCGC_2', 'GCGCA_2', 'CGCAC', 'GCACG', 'CACGG', 'ACGGC_1', 'CG
    GCG 1', 'GGCGC 1', 'GCGCA 1', 'CGCAA']
    GACGGCGGCGCACGGCGCAA
    True
    
## The k-1 De Bruijn Graph with k-mer edges



- We got the right answer, but we were lucky.
- There is a path in this graph that matches the Hamiltonian path that we found before


## What are the Differences?



## Choose a bigger k-mer

In [22]: $\quad \mathrm{M}=8$

```
target = "GACGGCGGCGCACGGCGCAA"
kmers = kmersUnique(target, k)
print(kmers)
nodes = sorted(set([code[:k-1] for code in kmers] + [code[1:k] for code in kmers]))
print(nodes)
G3 = Graph(nodes)
for code in kmers
    G3. addEdge(code[:k-1], code[1:k], code)
path = G3.eulerianPath()
print(path)
path = G3.eulerEdges(path)
print(path)
seq = path[0][0:k]
for kmer in path[1:]:
    seq += kmer[k-1]
print(seq)
print(seq == target)
['ACGGCGCA', 'ACGGCGGC', 'CACGGCGC', 'CGCACGGC', 'CGGCGCAA', 'CGGCGCAC', 'CGGCGGCG', 'GACGGCGG', 'GCACGGCG', 'GCGCACGG', 'GC
GGCGCA', 'GGCGCACG', 'GGCGGCGC']
['ACGGCGC', 'ACGGCGG', 'CACGGCG', 'CGCACGG', 'CGGCGCA', 'CGGCGGC', 'GACGGCG', 'GCACGGC', 'GCGCACG', 'GCGGCGC', 'GGCGCAA', 'G
GCGCAC', 'GGCGGCG']
[6, 1, 5, 12, 9, 4, 11, 8, 3, 7, 2, 0, 4, 10]
['GACGGCGG', 'ACGGCGGC', 'CGGCGGCG', 'GGCGGCGC', 'GCGGCGCA', 'CGGCGCAC', 'GGCGCACG', 'GCGCACGG', 'CGCACGGC', 'GCACGGCG', 'CA
CGGCGC', 'ACGGCGCA', 'CGGCGCAA']
GACGGCGGCGCACGGCGCAA
True
```


## Advantage of larger k-mers

- Making $k$ larger (8) eliminates the second choice of loops
- There are edges to choose from, but they all lead to the same path of vertices



## Applied to the Hamiltonian Solution

In [23]: $\quad \mathrm{M}=8$

```
target = "GACGGCGGCGCACGGCGCAA"
kmers = kmersUnique(target, k)
G4 = Graph(kmers)
for vsrc in kmers:
    for vdst in kmers:
        if (vsrc[1:k] == vdst[0:k-1]):
            G4.addEdge(vsrc,vdst)
path = G4.hamiltonianPathV2()
print(path)
seq = path[0][0:k]
for kmer in path[1:]:
    seq += kmer[k-1]
print(seq)
print(seq == target)
['GACGGCGG', 'ACGGCGGC', 'CGGCGGCG', 'GGCGGCGC', 'GCGGCGCA', 'CGGCGCAC', 'GGCGCACG', 'GCGCACGG', 'CGCACGGC', 'GCACGGCG', 'CA
CGGCGC', 'ACGGCGCA', 'CGGCGCAA']
GACGGCGGCGCACGGCGCAA
True
```


## Graph with 8-mers as vertices

- There is only one Hamiltonian path
- There are no repeated $k$-mers



## Assembly in Reality

- Problems with repeated k-mers
- We can't distinguish between repeated k-mers
- Recall we knew from our example that were \{2:ACGGC, 3:CGGCG, 2:GCGCA, 2:GGCGC\}
- Assembling path without repeats:

```
In [26]: M k = 5
```

```
target = "GACGGCGGCGCACGGCGCAA
```

target = "GACGGCGGCGCACGGCGCAA
kmers = set([target[i:i+k] for i in range(len(target) -k+1)])
kmers = set([target[i:i+k] for i in range(len(target) -k+1)])
nodes = sorted(set([code[:k-1] for code in kmers] + [code[1:k] for code in kmers]))
nodes = sorted(set([code[:k-1] for code in kmers] + [code[1:k] for code in kmers]))
G5 = Graph(nodes)
G5 = Graph(nodes)
for code in kmers
for code in kmers
G5.addEdge(code[:k-1], code[1:k], code)
G5.addEdge(code[:k-1], code[1:k], code)
print(sorted(G5.vertex.items()))
print(sorted(G5.vertex.items()))
print(G5.edge)
print(G5.edge)
[(0, 'ACGG'), (1, 'CACG'), (2, 'CGCA'), (3, 'CGGC'), (4, 'GACG'), (5, 'GCAA'), (6, 'GCAC'), (7, 'GCGC'), (8, 'GCGG'), (9, 'G
[(0, 'ACGG'), (1, 'CACG'), (2, 'CGCA'), (3, 'CGGC'), (4, 'GACG'), (5, 'GCAA'), (6, 'GCAC'), (7, 'GCGC'), (8, 'GCGG'), (9, 'G
GCG')]
GCG')]
[(9, 8), (3, 9), (1, 0), (4, 0), (6, 1), (8, 3), (0, 3), (2, 5), (7, 2), (2, 6), (9, 7)]

```
[(9, 8), (3, 9), (1, 0), (4, 0), (6, 1), (8, 3), (0, 3), (2, 5), (7, 2), (2, 6), (9, 7)]
```


## Resulting Graph with "unique" 5-mers as edges

- There is no single Euler Path
- But there are is a set of paths that covers all edges [ 'GACGGCG', 'GGCGGC', 'GGCGCA', 'CGCAA', 'CGCACGG' ]
- Extend a sequence from a node until you reach a node with an out-degree $\rightarrow$ in-degree
- Save these partially assembled subsequences, call them contigs
- Start new contigs following each out-going edge at these branching nodes



## Next assemble contigs

- Use a modified read-overlap graph to assemble these contigs
- Add edge-weights that indicate the amount of overlap

- Usually much smaller than the graph made from k-mers


## A Heavy Path

Find the heaviest path touching all vertices in this smaller graph

## GACGGCGGCGCACGGCGCAA <br> GACGGCG <br> GGCGGC <br> 4 <br> GGCGCA <br> 3 <br> CGCACGG 4 <br> GGCGCA 2 <br> CGCAA 4 <br> 17



## Discussion

- No simple single algorithm for assembling a real genome sequences
- Generally, an iterative task
- Choose a k-mer size, ideally such that no or few k-mers are repeated
- Assemble long paths (contigs) in the resulting graph
- Use these contigs, if they overlap suffciently, to assemble longer sequences
- Truely repetitive subsequences are a challenge
- Leads to repeated k-mers and loops in graphs in the problem areas
- Often we assemble the "shortest" version of a genome consistent with our k-mer set
- Things we've ignored
- Our k-mers are extracted from short read sequences that may contain errors
- Our short read set could be missing entire segments from the actual genome
- Our data actually supports 2 paths, one through the primary sequence, and a second through it again in reverse complement order.

